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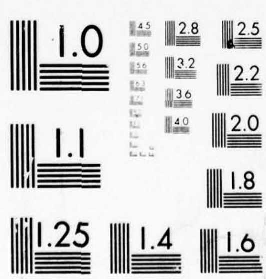

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THE EFFECT OF POLARIZATION DIVERSITY ON RECEIVING SYSTEMS.

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THE EFFECT OF POLARIZATION DIVERSITY ON RECEIVING SYSTEMS

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

Meir Lavi, B.S.E.E.

Maj IAF

Graduate Electrical Engineering

December 1977

Approved for public release; distribution unlimited.

Preface

The objective of writing this thesis was to establish a mathematical system model to enable Aeronautical Systems Division, Electronic Warfare Division to evaluate the effectiveness of polarization diversity technique used by a jammer against a radar system in a real environment.

I would like to express my deep appreciation to my advisor, Professor W. A. Davis for his continuing dedicated guidance. I owe my sincerest gratitude to my dear wife Zahava for her support and patience throughout my AFIT program.

Meir Lavi

This thesis was typed by Mrs. Hazel Gaudreau

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Abstract

The use of polarization diversity process requires the derivation of a mathematical system model to allow evaluating the effects of this process, employed by a jammer or by a threatened radar, upon jamming effectiveness. The system model consists of two orthogonal linear antennas controlled by a random process in amplitude and phase. The expected value of the received power due to interference is related to the autocorrelation of the received signal. The received power is obtained in the frequency domain from the spectral behavior of the total polarization diversity processes employed by both the jammer and radar and the spectral characteristics of the transmitting and receiving systems. The spectral behavior of the total polarization diversity process is constructed by the convolution between the polarization diversity processes of both the jammer and radar. The spectrum of the polarization diversity is spread in a convolution manner by the transmitter spectrum. This convolved spectrum contributes to the expected value of the received power only within the bandwidth of the receiving system. It is concluded that for several typical scenarios polarization diversity is an effective jamming technique.

THE EFFECT OF
POLARIZATION DIVERSITY
ON RECEIVING SYSTEMS

I. Introduction

Polarization diversity is the phenomenon of changing the polarization characteristics of an electromagnetic wave. The results of this phenomenon have been evaluated and implemented primarily in radio astronomy with some applications applied to radar systems. However, the evaluation of the effects of polarization diversity upon reception, presented in radio astronomy literature, has been limited to the case where only the transmitted polarization is varying while the polarization state of the receiving antenna is kept fixed. The objective of this thesis is to derive a mathematical model which is used as a tool for evaluating the effects of polarization diversity, employed by a jammer or by a threatened radar system. Once the mathematical model has been derived, it is shown how one may go about optimizing the likelihood of reception by the radar system.

Two different categories of polarization diversity are presented. The first one is formed implicitly by one or more radar systems with different fixed polarization states, by which the equivalent polarization state seen by a jammer becomes a random variable. The second one is when a random process is employed explicitly by the radar system in order to reduce the effectiveness of polarization jamming. For both categories, one should consider the statistical behavior of the polarization processes when evaluating the effects of those processes upon the likelihood of reception.

The derivation of the time domain model in Chapter II is done using the representation of a polarization state given by the Stokes parameters or the Poincare sphere and using the theory of receiving partially polarized electromagnetic waves as presented in the radio astronomy literature. The definition of the Stokes parameters given in the radio astronomy literature is revised to enable its usage in a more general situation where the receiver, as well as the transmitter, employs a time varied polarization diversity. Implementation considerations are used in determining the system model used for polarization diversity. The general transfer function of the receiving-transmitting system model in the time domain is derived. This transfer function is reduced to the function given in the radio astronomy literature once some assumptions are made. Those assumptions are listed in this chapter.

In Chapter III, the system model derived in Chapter II is transformed from the time domain representation into a statistical representation. The averaging process which is used for evaluating the likelihood of reception is done by the ensemble average operation rather than by a time average operation. Since the expected value of the received power is related to the autocorrelation of the received voltage, the whole system is described in a statistical second moment representation. When statistically stationary is assumed, the Fourier transform of this representation leads to frequency domain representation. The effects of the polarization diversity processes as well as of the frequency characteristics of the jammer and the radar systems appear in the equation of the average received power in the frequency domain. The power equation is simplified for various classes of assumptions.

In Chapter IV, several feasible scenarios are evaluated with respect to the frequency domain equation derived in Chapter III. Results are obtained for several cases both analytically and numerically.

II. The Time Dependence of the Received Power on Polarization Diversity

The dependence of the power, received by a receiving system, on time variation polarization diversity which is employed by a transmitter, has been derived in the radio astronomy literature (Ref 1; Ref 2). This dependence is first reviewed in this chapter, where it has been assumed that the receiving system has no variation in polarization. The dependence of the received power on the varying polarization, which is employed by both systems, receiver and transmitter, is then included.

The polarization diversity, discussed in the radio astronomy literature was created by nature and there has been no need to discuss the ways to implement such a phenomenon. However, since this study is dealing with a man made polarization diversity system, a feasible model will be presented. Thus, the derivation of the effect of the polarization variations will be related to that model.

Polarization State

A monochromatic plane wave can be expressed analytically by its electric field as

$$\bar{E}(\bar{r},t) = \mathrm{Re}\left[\bar{E}_o e^{j(\omega t - \bar{k}\cdot\bar{r})}\right] \tag{1}$$

where $\bar{k}$ is the propagation vector defined by $\bar{k} = k\hat{k}$.

Since the wave is plane therefore $\overline{E}_o$ is perpendicular to the propagation direction $\hat{k}$, that is, $\hat{k}\cdot\overline{E}_o = 0$. Without loss of generality $\hat{k}$ can be chosen to be parallel to the r axis in the spherical coordinate system r, θ, and ϕ. $\overline{E}_o$ can be written then as

$$\overline{E}_o = \hat{\theta}A + \hat{\phi}A_2$$

$$= \hat{\theta}|A_1|e^{j\alpha_1} + \hat{\phi}|A_2|e^{j\alpha_2} \tag{2}$$

where $\hat{\theta}$ and $\hat{\phi}$ are the unit vectors along the θ and ϕ axes, respectively, and where the magnitudes $|A_1|$ and $|A_2|$, as well as the phases α_1 and α_2, are real constants. The reason for choosing the spherical coordinate system is explained in Appendix A.

It follows from Equations (1) and (2) that the spherical components of $\overline{E}(r,t)$ are given by the real expressions

$$E_\theta = |A_1|\cos(\delta + \alpha_1) \tag{3}$$

$$E_\phi = |A_2|\cos(\delta + \alpha_2) \tag{4}$$

where

$$\delta = \omega t - kr \tag{5}$$

Eliminating δ from these expressions, one obtains the following equation (Ref 3:24-25)

$$\left(\frac{E_\theta}{|A_1|}\right)^2 + \left(\frac{E_\phi}{|A_2|}\right)^2 - 2\,\frac{E_\theta}{|A_1|}\,\frac{E_\phi}{|A_2|}\cos\alpha = \sin^2\alpha \tag{6}$$

where

$$\alpha = \alpha_1 - \alpha_2 \tag{7}$$

When taking E_θ and E_ϕ as coordinate axes, it is seen that Equation (6) represents an ellipse whose center is located at the origin $E_\theta = E_\phi = 0$. This ellipse is illustrated in Figure 1. The polarization state of the plane wave is defined by this ellipse and by the direction of the rotation of $\bar{E}(r)$ along the ellipse. The direction of the rotation depends on α. The ellipse is set by the following three independent parameters $|A_1|$, $|A_2|$, and α.

The ellipse can be represented by a different set of parameters which consists of the semimajor and semiminor axes of the ellipse, denoted by a and b respectively, and the orientation angle ψ between the major axis of the ellipse and the θ axis of the spherical coordinate system. This set is illustrated in Figure 1. The relations between the two sets are given by the following equations (Ref 3:26-27)

$$a^2 + b^2 = |A_1|^2 + |A_2|^2 \tag{8}$$

and

$$\tan 2\psi = \frac{2|A_1||A_2|}{|A_1|^2 - |A_2|^2}\cos\alpha, \quad 0 \leq \psi < \pi \tag{9}$$

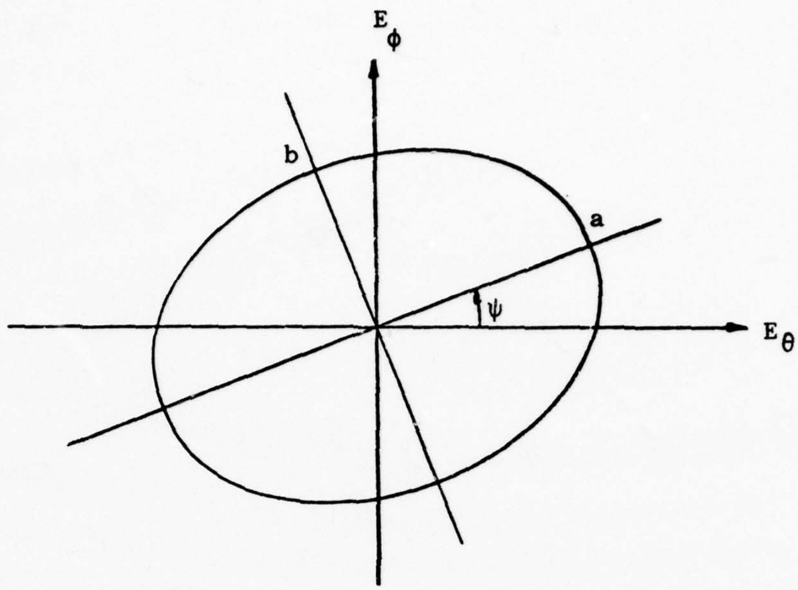

Figure 1. Elliptic Representation of Polarization State

which were derived by equating the two different representations of the ellipse.

The polarization state of a plane wave may be defined also by a set of parameters which are known as Stokes parameters (Ref 4). The Stokes parameters of a monochromatic plane wave are defined by the following four quantities.

$$S_o = (|A_1|^2 + |A_2|^2)\frac{1}{Z_o} \tag{10}$$

$$S_1 = (|A_1|^2 - |A_2|^2)\frac{1}{Z_o} \tag{11}$$

$$S_2 = (2|A_1||A_2| \cos\alpha)\frac{1}{Z_o} \tag{12}$$

$$S_3 = (2|A_1||A_2| \sin\alpha)\frac{1}{Z_o} \tag{13}$$

where Z_o is the intrinsic impedance of an isotropic lossless medium.

Since these parameters are related by the identity

$$S_o^2 = S_1^2 + S_2^2 + S_3^2 \tag{14}$$

only three of the four are independent. S_o is equal to the Poynting vector of the wave since, when using rms quantities, the poynting vector is

$$\begin{aligned} \overline{S} &= \overline{E} \times \overline{H}^* \\ &= \left|\overline{E}_o \cdot \overline{E}_o^*\right| \hat{r} \frac{1}{Z_o} \\ &= \frac{1}{Z_o}\left||A_1|^2 + |A_2|^2\right| \hat{r} \end{aligned} \tag{15}$$

and the magnitude will be

$$\begin{aligned} S &= \frac{1}{Z_o}(|A_1|^2 + |A_2|^2) \\ &= S_o \end{aligned} \tag{16}$$

The Stokes parameters set is usually normalized such that

$$\begin{aligned} s_o &= \frac{S_o}{S_o} \\ &= 1 \end{aligned} \tag{17}$$

and

$$s_i = \frac{S_i}{S_o} \quad , \quad i = 1,2,3 \tag{18}$$

The Stokes parameters are expressed as a [4x1] vector

$$S[s_i] = S \begin{bmatrix} s_o \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} \quad , \quad i = 0,1,2,3 \tag{19}$$

where S is the Poynting vector in watts $-m^{-2}$. The significance of the Stokes parameters can be understood as the representation of the characteristics of the power of the wave.

The effective aperture of a receiving antenna also can be represented by the Stokes parameters of the wave radiated by the antenna with the antenna transmitting. Thus the set will be noted by (Ref 5)

$$A_e[a_i] \quad , \quad i = 0,1,2,3 \tag{20}$$

where the effective aperture is

$$A_e = \frac{\lambda^2 G}{4\pi} \tag{21}$$

assuming a matched antenna, and

$$a_o = 1 \tag{22}$$

$$a_1 = \frac{|B_1|^2 - |B_2|^2}{Z_o S_a} \qquad (23)$$

$$a_2 = \frac{2}{Z_o S_a} |B_1||B_2| \cos(-\beta) \qquad (24)$$

$$a_3 = \frac{2}{Z_o S_a} |B_1||B_2| \sin(-\beta) \qquad (25)$$

where S_a is the magnitude of the poynting vector or the power density of the wave radiated by the antenna. The reason for choosing a minus sign in front of β is the direction of the received wave which is opposite to the direction of the transmitted wave by the antenna, by which the Stokes parameters have been defined.

Alternatively, the Stokes parameters can be written in terms of the orientation angle ψ and the ellipticity angle χ as follows;

$$s_1 = s_o \cos 2\chi \cos 2\psi \qquad (26)$$

$$s_2 = s_o \cos 2\chi \sin 2\psi \qquad (27)$$

$$s_3 = s_o \sin 2\chi \qquad (28)$$

s_1, s_2, and s_3 can be interpreted as the cartesian coordinates of a point on a sphere of radius s_o where the longitude and latitude of the point are 2ψ and 2χ respectively. The ellipticity angle χ is defined by

$$\tan\chi = \pm \frac{b}{a}, \quad -\pi/4 \le \chi \le \pi/4 \qquad (29)$$

where

$$\sin 2\chi = \frac{2|A_1||A_2|}{|A_1|^2+|A_2|^2}\sin\alpha = \frac{S_3}{S_o} = s_3 \tag{30}$$

This sphere is called the Poincare sphere and it is illustrated in Figure 2. A point on the sphere represents a polarization state. Linear polarization corresponds to the equator of the sphere or to zero ellipticity angle χ. Circular polarization corresponds to the poles of the sphere or to $2\chi = \pm\pi/2$ where the plus sign denotes a right-handed circular polarization and the minus sign a left-handed circular polarization. It is convenient to describe the polarization state by either one of the two sets of angles ψ and χ or α and γ where γ is defined as

$$\gamma = \arctan\frac{|A_2|}{|A_1|}, \quad 0 \leq \gamma \leq \pi/2 \tag{31}$$

The relations between the two sets are given by (Ref 3:27)

$$\begin{aligned} \tan 2\psi &= \tan 2\gamma\cos\alpha \\ \sin 2\chi &= \sin 2\gamma\sin\alpha \end{aligned} \tag{32}$$

The two sets are illustrated in Figure 2.

In summary, the polarization state has been defined by its elliptic representation and by two parametric representations, Stokes parameters and Poincare sphere. The representation by Stokes parameters is used next to evaluate the received power.

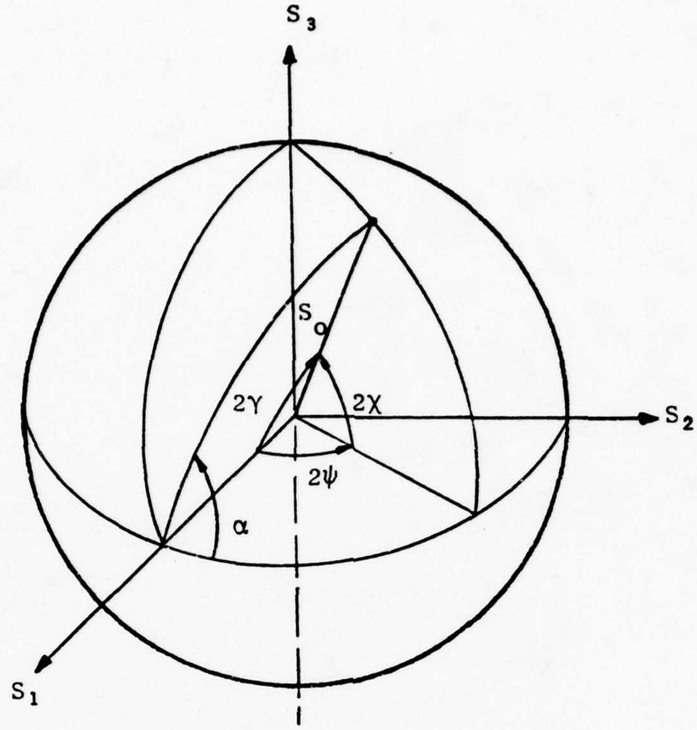

Figure 2. Poincare Sphere

<u>The Effect of Polarization Diversity on the Reception of an Antenna</u>

It has been shown by Ko (1962) that the power P available from an antenna whose effective aperture is $A_e[a_i]$, when a wave of polarization $S[s_i]$ is incident upon it, is given by (Ref 5)

$$P = \tfrac{1}{2} S A_e [\tilde{a}_i][s_i] = \tfrac{1}{2} S A_e \sum_{i=0}^{3} a_i s_i \tag{33}$$

where $[\tilde{a}_i]$ is the transpose of $[a_i]$. The complete derivation of Equation (33) is given in Appendix A. The last three Stokes parameters are representing the three components of a radius vector of the Poincare sphere. Thus, the radius vector of the incident wave will be defined as

$$\hat{s} = s_1 \hat{s}_1 + s_2 \hat{s}_2 + s_3 \hat{s}_3 \tag{34}$$

and the radius vector of the antenna will be defined as

$$\hat{a} = a_1\hat{a}_1 + a_2\hat{a}_2 + a_3\hat{a}_3 \quad (35)$$

The dot product of the two vectors could be replaced by the cosine of the angle ε between the vectors. Thus

$$\hat{s}\cdot\hat{a} = s_1a_1 + s_2a_2 + s_3a_3 = \cos\varepsilon \quad (36)$$

and then

$$P = \tfrac{1}{2}SA_e(1 + \cos\varepsilon) \quad (37)$$

As illustrated in Figure 3, the maximum available power is reached when $\varepsilon = \phi$ or 2π, and zero power is received when $\varepsilon = \pi$.

A quasi-monochromatic plane wave, which consists of a superposition of a large number of statistically independent waves with a variety of polarization states, is said to be partially polarized. The frequency bandwidth Δf of such a wave is assumed to be very small compared to the mean frequency f. The electric field of the quasi-monochromatic plane wave may be uniquely represented by the analytic form

$$\bar{E}(r,t) = \mathrm{Re}\left[\bar{E}_o(t)e^{j(\omega t - \bar{k}\cdot\bar{r})}\right] \quad (38)$$

where ω denotes the average value of the frequency. Since the bandwidth of the field is very narrow, $\bar{E}_o(t)$ will be a slowly varying function of time compared to $e^{j\omega t}$. $\bar{E}_o(t)$ can be written in the form

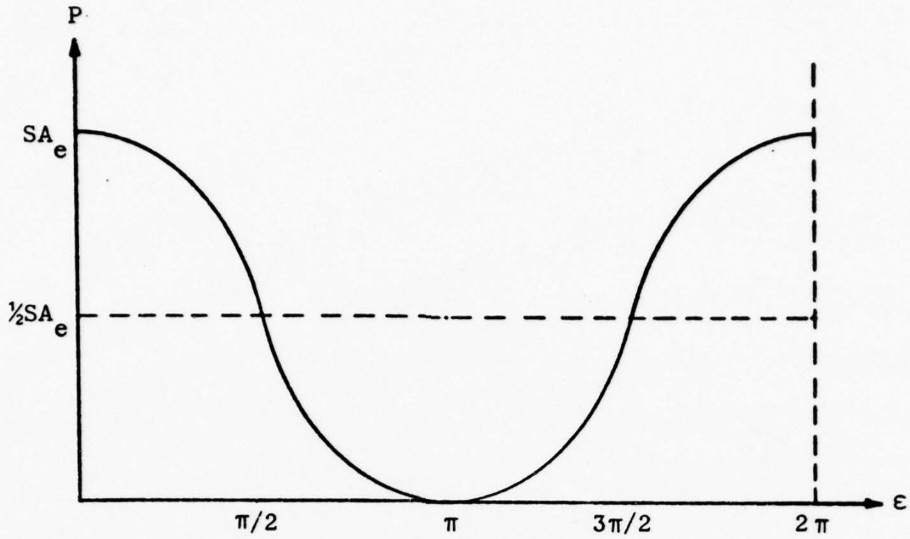

Figure 3. The Received Power versus ε

$$\overline{E}_o(t) = \hat{\theta}|A_2(t)|e^{j\alpha_1(t)} + \hat{\phi}|A_2(t)|e^{j\alpha_2(t)} \tag{39}$$

where the magnitudes $|A_1(t)|$, $|A_2(t)|$ and the phases $\alpha_1(t)$, $\alpha_2(t)$ are slowly varying functions of time. The spherical components of $\overline{E}(\overline{r},t)$ are given by

$$E_\theta = |A_1(t)|\cos(\delta + \alpha_1(t)) \tag{40}$$

$$E_\phi = |A_2(t)|\cos(\delta + \alpha_1(t) - \alpha(t)) \tag{41}$$

$$E_r = 0 \tag{42}$$

where

$$\alpha(t) = \alpha_1(t) - \alpha_2(t) \tag{43}$$

and $\delta(t) = \omega t - \overline{k}\cdot\overline{r}$

Although the magnitudes and phases of the components of the quasi-monochromatic plane wave are irregularly varying functions of time, certain correlations may exist among them. These correlations determine the Stoke parameters and consequently the polarization state of the wave. The Stokes parameters of the quasi-monochromatic plane wave have been defined as the time-averaged quantities (Ref 1)

$$S_o = \frac{1}{Z_o} [<|A_1(t)|^2> + <|A_2(t)|^2>] \tag{44}$$

$$S_1 = \frac{1}{Z_o} [<|A_1(t)|^2> - <|A_2(t)|^2>] \tag{45}$$

$$S_2 = \frac{2}{Z_o} <|A_1(t)||A_2(t)|\cos\alpha(t)> \tag{46}$$

$$S_3 = \frac{2}{Z_o} <|A_1(t)||A_2(t)|\sin\alpha(t)> \tag{47}$$

where

$$<x(t)> = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)dt \tag{48}$$

It has been shown that the Stokes parameters of this plane wave satisfy the relation (Ref 2:120)

$$S_o^2 \geq S_1^2 + S_2^2 + S_3^2 \tag{49}$$

By dividing through by S_o, the normalized Stokes parameters vector will be then

$$[s_i] \; , \quad i = 0,1,2,3 \tag{50}$$

where $s_o = 1$. The degree of polarization of the quasi-monochromatic plane wave is defined as

$$d = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_o}$$

$$= \sqrt{s_1^2 + s_2^2 + s_3^2} \tag{51}$$

It follows that for a partially polarized wave the Stokes parameters set can be represented by

$$s_o = 1 \tag{52}$$

$$s_1 = d \cos 2\chi \cos 2\psi \tag{53}$$

$$s_2 = d \cos 2\chi \sin 2\psi \tag{54}$$

$$s_3 = d \sin 2\chi \tag{55}$$

The definition of the Stokes parameters which has been given in the radio astronomy literature, can be applied only when assuming no time variation in the receiving antenna system or in the receiving system itself. However, for a complete view, one should consider a general case where there are variations in time in the receiving antenna system and in the receiving system itself. This consideration leads to the definition of the Stokes parameters as instantaneous functions of time while the averaging process is performed on the instantaneous power function. Thus, without taking into consideration the processing of the receiving

system, the average received power will be

$$P = \tfrac{1}{2} \langle SA_e [\tilde{a}_i][s_i] \rangle$$

$$= \tfrac{1}{2} \langle SA_e \sum_{i=0}^{3} a_i s_i \rangle \qquad (56)$$

where the averaging process will be defined either by Equation (48) or as a statistical average, depending whether the variations are deterministic or random, respectively. The averaging process will be discussed in detail in Chapter III.

When defining the Stokes parameters by the θ and ϕ components, there is no consideration how one would go about implementing the plane wave and the receiving antenna system components such that the amplitudes, A_i's and B_i's, can be controlled independently. The necessity of having independent control of the amplitudes results from the objective of this study, namely, developing an optimized polarization diversity procedure. In the following, a feasible implementation of a polarization diversity model will be considered.

Polarization Diversity System Model

A wave of arbitrary polarization may be produced by combining the waves radiated by a pair of crossed-dipole antennas. If the dipoles are aligned parallel to the u and v axes of an arbitrary coordinate system (u,v,w), then the components of the wave radiated in the w direction will have the form

$$E_u = |A_1| e^{j\alpha_1} \qquad (57)$$

$$E_v = |A_2| e^{j\alpha_2} \qquad (58)$$

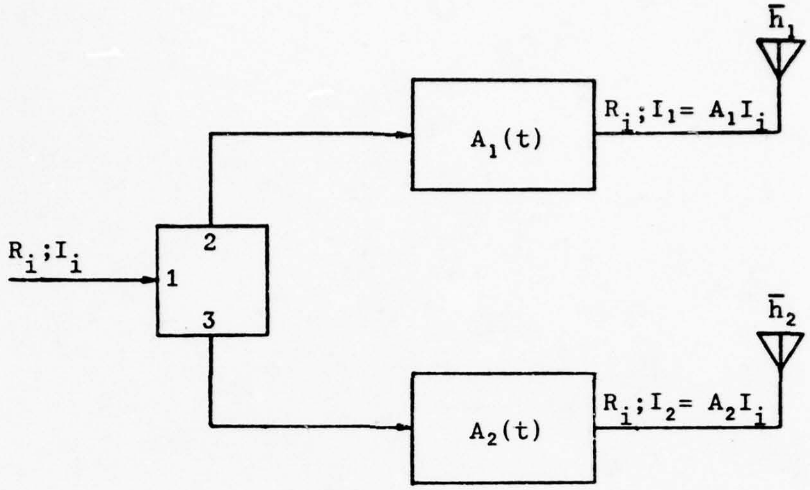

Figure 4. Polarization Diversity System Model

in which $|A_1|$ and $|A_2|$ are positive real magnitude factors, and α_1 and α_2 are the phase angles associated with the field components. The relative magnitudes and the phases of the field components may be adjusted to give any desired polarization.

The system which will be used for evaluating the effects of polarization diversity upon the received power consists of two orthogonal linear antenna channels connected to a common source through an impedance matched T adapter which is called a power splitter. This system will be the model for both functions, receiving and transmitting. The system is illustrated in Figure 4. Each antenna channel consists of a linear antenna and a magnitude and phase controller A_i which is controlling the input or output current of the antenna, depending whether the system is transmitting or receiving, respectively.

The characteristic matrix of the power splitter will be

$$\begin{bmatrix} V_1^r \\ V_2^r \\ V_3^r \end{bmatrix} = \begin{bmatrix} \zeta & \sigma/\sqrt{2} & \sigma/\sqrt{2} \\ \sigma/\sqrt{2} & \zeta & 0 \\ \sigma/\sqrt{2} & 0 & \zeta \end{bmatrix} \begin{bmatrix} V_1^i \\ V_2^i \\ V_3^i \end{bmatrix} \tag{59}$$

where r denotes the reflected voltages and i denotes the incident voltages at the associated ports. ζ denotes the reflection coefficient and σ denotes the loss coefficient. When the splitter is matched then $\zeta = 0$ and $\sigma = 1$.

The effective height of the antenna system is obtained from Appendix A as

$$\bar{h} = \int_{v'} \frac{\bar{I}_a}{I_i} e^{jk\hat{r}\cdot\bar{r}'} dv'$$

$$= \left(\frac{I_1}{I_i}\right) \bar{h}_1 + \left(\frac{I_2}{I_i}\right) \bar{h}_2 \tag{60}$$

or

$$\bar{h} = |A_1| e^{j\alpha_1}\bar{h}_1 + |A_2| e^{j\alpha_2}\bar{h}_2 \tag{61}$$

The radiated power transmitted by this antenna system is derived in Appendix A and is given by

$$P_{rad} = Z_o \left(\frac{|I_i|}{2\lambda}\right)^2 \int_{\Omega} |\bar{h}\times\hat{r}|^2 d\Omega \tag{62}$$

where $d\Omega = \sin\theta d\theta d\phi$ and the integration is done over a sphere.

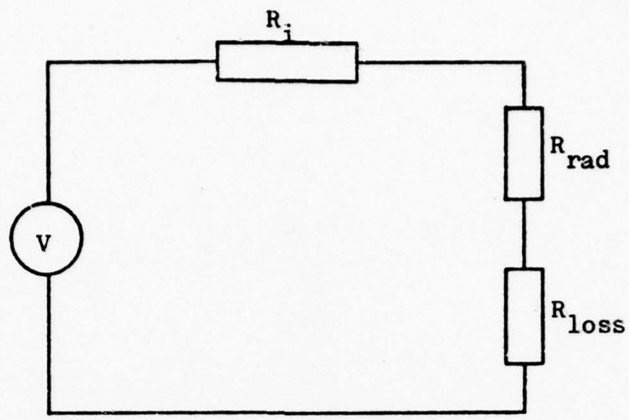

Figure 5. Equivalent Circuit of a Transmitting Antenna

Due to matching considerations one should keep the total radiating resistance constant. However, since the radiating resistance of this model is a function of the A_i's, there has to be a loss resistance which will compensate the changes in the radiating resistance. This configuration is illustrated in Figure 5. Thus, when the antenna is matched, $X_i = - X_{rad}$,

$$R_i = R_{rad} + R_{loss} \tag{63}$$

and

$$\begin{aligned} P_t &= |I_i|^2 (R_{rad} + R_{loss}) \\ &= |I_1|^2 R_i + |I_2|^2 R_i + |I_i|^2 R_{loss} \\ &= |I_i|^2 [R_i(|A_1|^2 + |A_2|^2) + R_{loss}] \\ &= |I_i|^2 R_i \end{aligned} \tag{64}$$

Therefore

$$R_{rad} = R_i(|A_1|^2+|A_2|^2) \tag{65}$$

and

$$R_{loss} = R_i(1-|A_1|^2-|A_2|^2) \tag{66}$$

where R_i is given by Equation (A-37) as

$$\begin{aligned} R_i &= \frac{Z_o}{4\lambda^2} \int_\Omega |\bar{h}_1 x \hat{r}|^2 d\Omega \\ &= \frac{Z_o}{4\lambda^2} \int_\Omega |\bar{h}_2 x \hat{r}|^2 d\Omega \end{aligned} \tag{67}$$

and R_{loss} represents the power dissipated in the antenna system. It has been assumed that the two antennas have identical input impedances. One can verify that the sum of the radiated powers of the two channels given below is equal to the radiated power of the total system assuming that the two antennas have orthogonally polarized antenna patterns.

$$\begin{aligned} P_{t1} + P_{t2} &= Z_o \left(\frac{|I_1|}{2\lambda}\right)^2 \int_\Omega |\bar{h}_1 x \hat{r}|^2 d\Omega \\ &\quad + Z_o \left(\frac{|I_2|}{2\lambda}\right)^2 \int_\Omega |\bar{h}_2 x \hat{r}|^2 d\Omega \\ &= Z_o \left(\frac{|I_i|}{2\lambda}\right)^2 (|A_1|^2+|A_2|^2) \int_\Omega |\bar{h}_1 x \hat{r}|^2 d\Omega \end{aligned} \tag{68}$$

since

$$\int_{\Omega} |\bar{h}x\hat{r}|^2 d\Omega = \int_{\Omega} |(A_1\bar{h}_1 + A_2\bar{h}_2) \times \hat{r}|^2 d\Omega$$

$$= \int_{\Omega} (A_1\bar{h}_1 x\hat{r} + A_2\bar{h}_2 x\hat{r}) \cdot (A_1\bar{h}_1 x\hat{r} + A_2\bar{h}_2 x\hat{r})^* d\Omega$$

$$= |A_1|^2 \int_{\Omega} |\bar{h}_1 x\hat{r}|^2 d\Omega + |A_2|^2 \int_{\Omega} |\bar{h}_2 x\hat{r}|^2 d\Omega$$

$$= \int_{\Omega} (|A_1|^2 + |A_2|^2) \, |\bar{h}_1 x\hat{r}|^2 d\Omega \tag{69}$$

and

$$\bar{h}_1 x\hat{r} \cdot \bar{h}_2 x\hat{r} = 0 \tag{70}$$

due to the pattern orthogonality.

Assuming the same model for both the receiving and the transmitting systems, one obtains the total system as illustrated in Figure 6. The received signal $v_o(t)$ will be formulated by using Equation (A-41) and convolving $v_r(t)$ with $g_r(t)$ to obtain

$$v_o(t) = g_r(t) * [\bar{E}_o(t - R/c) \cdot \bar{h}_r(t)] \tag{71}$$

where $h_r(t)$ is defined as

$$\bar{h}_r(t) = B_1(t)\bar{h}_{r1} + B_2(t)\bar{h}_{r2} \tag{72}$$

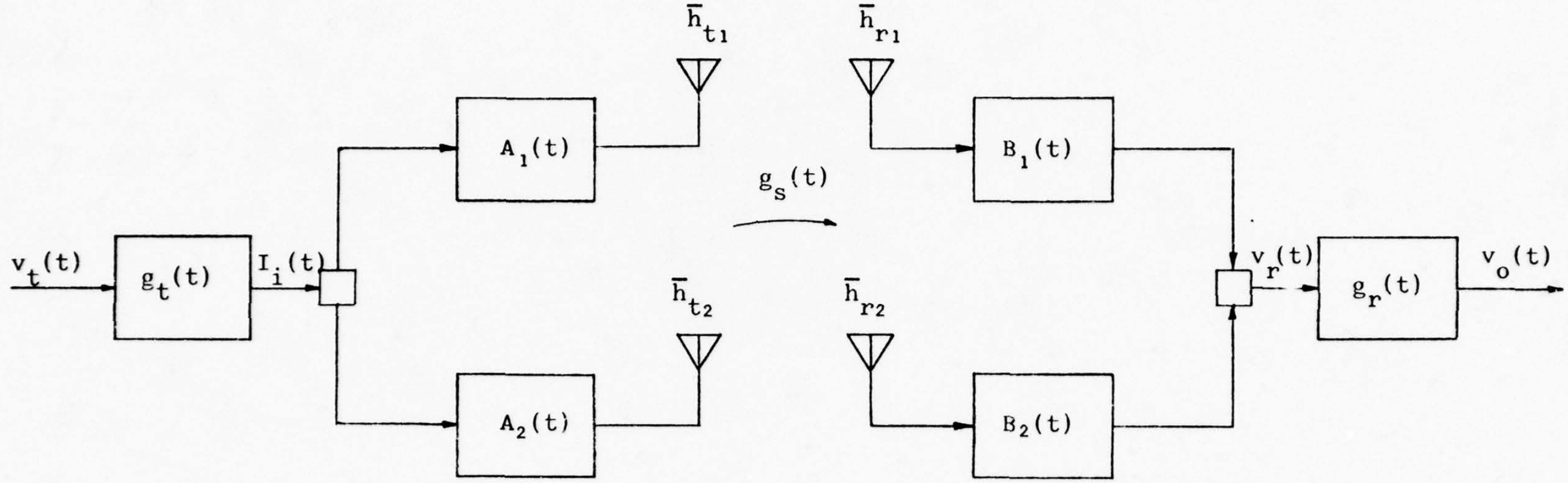

Figure 6. Receiving-Transmitting Model System

$\bar{E}_o(t)$ is obtained from Equations (A-10) and (A-23) by considering $\bar{E}_o$ in the frequency domain as

$$\bar{E}_o(\omega) = -\frac{j\mu(\omega + \omega_o)}{4\pi R} e^{-j(\omega + \omega_o)R/c} \bar{N}_t'(\omega) \tag{73}$$

where $\bar{N}_t'(\omega)$ is the radiation vector in frequency domain and $\bar{N}_t'(\omega) = -\hat{r}x\hat{r}x\bar{N}(\omega)$ while $\bar{N}(\omega)$ is defined in Appendix A by Equation (A-7).

Since $\omega << \omega_o$,

$$\bar{E}_o(\omega) \simeq -\frac{j\omega_o\mu}{4\pi R} e^{-j(\omega + \omega_o)R/c} \bar{N}_t'(\omega) \tag{74}$$

and

$$\bar{E}_o(t) \simeq -\frac{j\omega_o\mu}{4\pi R} e^{-j\omega_o R/c} \delta(t - R/c) * \bar{N}_t'(t)$$

$$= -\frac{j\omega_o\mu}{4\pi R} e^{-j\omega_o R/c} I_i(t - R/c)\bar{h}_t'(t - R/c) \tag{75}$$

where

$$I_i(t) = v_t(t) * g_t(t) \tag{76}$$

Due to the pattern orthogonality one can approximate $\bar{h}_t'(t)$ by $\bar{h}_t$ even for angles far off beam center for many such antenna types. Thus,

$$\bar{h}_t'(t) \simeq A_1(t)\bar{h}_{t1} + A_2(t)\bar{h}_{t2} \tag{77}$$

to give

$$v_o(t) = g_r(t) * \left[\left[\left(-\frac{j\omega_o\mu}{4\pi R} e^{-j\omega_o R/c} \left[v_t(t-R/c) * g_t(t)\right]\right) \bar{h}_t(t-R/c) \cdot \bar{h}_r(t)\right]\right] \tag{78}$$

The $\bar{h}_{ri}$'s and the $\bar{h}_{ti}$'s are assumed to be constant, with respect to frequency and time, in the close neighborhood of ω_o. By using Equations (72) and (77), the quantity $\bar{h}_t \cdot \bar{h}_r$ is given by

$$\begin{aligned}
\bar{h}_t(t-R/c) \cdot \bar{h}_t(t) &= [A_1(t-R/c)\bar{h}_{t1} + A_2(t-R/c)\bar{h}_{t2}] \cdot [B_1(t)\bar{h}_{r1} + B_2(t)\bar{h}_{r2}] \\
&= A_1(t-R/c)B_1(t)(\bar{h}_{t1} \cdot \bar{h}_{r1}) \\
&\quad + A_1(t-R/c)B_2(t)(\bar{h}_{t1} \cdot \bar{h}_{r2}) \\
&\quad + A_2(t-R/c)B_1(t)(\bar{h}_{t2} \cdot \bar{h}_{r1}) \\
&\quad + A_2(t-R/c)B_2(t)(\bar{h}_{t2} \cdot \bar{h}_{r2})
\end{aligned} \tag{79}$$

The received power is given in Appendix A by Equation (A-40) as

$$P_r(t) = \frac{v(t)v(t)^*}{4R_L} \tag{80}$$

As illustrated in Figure 7, the maximum received power will be achieved when

$$\begin{aligned}
R_L &= R_{rad} + R_{loss} \\
&= R_i
\end{aligned} \tag{81}$$

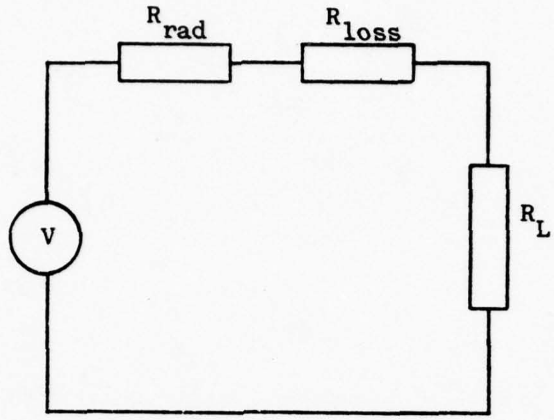

Figure 7. Equivalent Circuit of a Receiving Antenna

where

$$R_{rad} = R_i(|B_1|^2 + |B_2|^2) \tag{82}$$

$$R_{loss} = R_i[1 - (|B_1|^2 + |B_2|^2)] \tag{83}$$

and $v(t)$ is measured at the output of the receiving antenna system. R_i is defined by Equation (67). Hence, the received power is related to $v_o(t)v_o^*(t)$ where

$$v_o(t)v_o^*(t) = \left(\frac{\omega\mu}{4\pi R}\right)^2 \left|g_r(t)*[(v_t(t-R/c)*g_t(t))\bar{h}_t(t-R/c)\cdot\bar{h}_r(t)]\right|^2 \tag{84}$$

This is the general transfer function of the received power of the transmitting-receiving system.

It can be shown that Equation (84) reduces to the representation of the received power in terms of the Stokes parameter sets of the transmitter and the receiver as a special case.

In the radio astronomy literature $v_t(t)$ was assumed to be constant, $g_t(t)$ was assumed to be $C\delta(t)$ where C compensates for the change of units between $v_t(t)$ and $I_i(t)$, $g_r(t)$ was assumed to be $\delta(t)$, and the antenna was assumed to be lossless. Thus

$$P_r(t) = \frac{v_o(t)v_o^*(t)}{4R_{rad}}$$

$$= \frac{1}{4R_{rad}}\left(\frac{\omega\mu}{4\pi R}\right)^2 |I_i|^2 |\bar{h}_t(t-R/c)\cdot\bar{h}_r(t)|^2 \tag{85}$$

In the case where

$$\bar{h}_{t1} = \hat{\theta}h_{t1}(\theta,\phi)$$

$$\bar{h}_{t2} = \hat{\phi}h_{t2}(\theta,\phi)$$

$$\bar{h}_{r1} = \hat{\theta}h_{r1}(\theta,\phi)$$

$$\bar{h}_{r2} = \hat{\phi}h_{r2}(\theta,\phi)$$

and by including the functions of θ and ϕ in the respective A_i's and B_i's

$$|\bar{h}_t(t-R/c)\cdot\bar{h}_r(t)|^2 = |A_1(t-R/c)B_1(t) + A_2(t-R/c)B_2(t)|^2$$

$$= |A_1(t-R/c)B_1(t)|^2 + |A_2(t-R/c)B_2(t)|^2$$

$$+ 2\mathrm{Re}[A_1(t-R/c)A_2^*(t-R/c)B_1(t)B_2^*(t)] \tag{86}$$

or

$$|\bar{h}_r(t-R/c)\cdot\bar{h}_r(t)|^2 = \tfrac{1}{2}[|A_1(t-R/c)|^2+|A_2(t-R/c)|^2][|B_1(t)|^2+|B_2(t)|^2]$$

$$+ \tfrac{1}{2}[|A_1(t-R/c)|^2-|A_2(t-R/c)|^2][|B_1(t)|^2-|B_2(t)|^2]$$

$$+ 2|A_1(t-R/c)||A_2(t-R/c)||B_1(t)||B_2(t)|\cos(\alpha-\beta) \tag{87}$$

where

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \tag{88}$$

In the definition of the Stokes parameters of the incident wave, B_i represents the electric field $\bar{E}_i$ instead of the effective height $\bar{h}_{ti}$. Thus, by using Equation (A-23) and defining the field as

$$\bar{E}_1 = -\frac{j\omega\mu}{4\pi R} e^{-jkR} I_i A\,(t)\bar{h}_{t1}$$

$$= A_1'(t)\bar{h}_{t1} \tag{89}$$

and

$$\bar{E}_2 = -\frac{j\omega\mu}{4\pi R} e^{-jkR} I_i A\,(t)\bar{h}_{t2}$$

$$= A_2'(t)\bar{h}_{t2} \tag{90}$$

the received power will be

$$P_r = \frac{a_o}{4R_{rad}} \tfrac{1}{2} \sum_{i=0}^{3} S_i a_i \tag{91}$$

By using Equations (A-47) and (A-49) one can show that the received power will have the form

$$P_r = \tfrac{1}{2} A_e S \sum_{i=0}^{3} s_i a_i \tag{92}$$

which is equivalent to Equation (33).

III. The Average Received Power as a Function of the Polarization Diversity Random Processes

The system which was illustrated in Figure 5 can be represented as a transfer system where $v_t(t)$, $\bar{h}_t(t)$, and $\bar{h}_r(t)$ are inputs and $v_o(t)$ is the output. This system is illustrated in Figure 8. The space transfer function, $g_s(t)$, represents the attenuation and the phase changes in the signal due to the distance R. Thus

$$G_s(\omega) = -\frac{j(\omega+\omega_o)\mu}{4\pi R} e^{-j(\omega+\omega_o)R/c} \tag{93}$$

where ω_o is the carrier frequency,

$$(\omega+\omega_o)/c = k + k_o \tag{94}$$

and

$$g_s(t) = -\left[\frac{\mu\delta'(t-R/c)}{4\pi R} + \frac{j\omega_o\mu\delta(t-R/c)}{4\pi R}\right] e^{-jk_oR} \tag{95}$$

Since $\omega_o \gg \omega$, the derivative term can be neglected and

$$\begin{aligned} g_s(t) &\simeq -\frac{j\omega_o\mu\delta(t-R/c)}{4\pi R} e^{-jk_oR} \\ &= Q(R)\delta(t-R/c) \end{aligned} \tag{96}$$

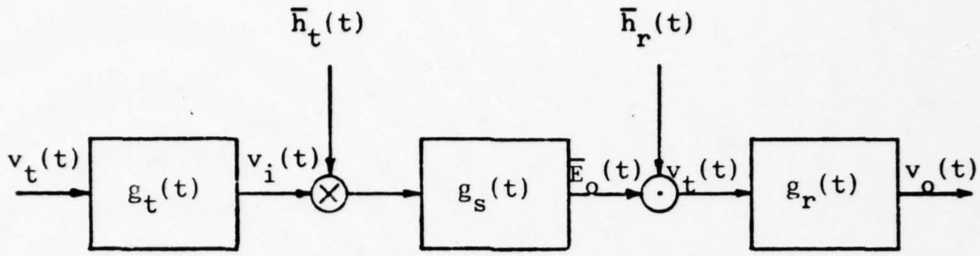

Figure 8. Transfer Function Representation

As it was stated in Chapter II, the approach for evaluating the received power will be by averaging. In the most general case $\bar{h}_t(t)$ and $\bar{h}_r(t)$ are assumed to be random processes. One could evaluate the time average received power by assuming ergodicity and evaluating the expected value of the power. In any case, from a statistical point of view one may look for the expected value of the power since it will tell more about the likelihood of reception at any time than would the time average operation.

The received power per unit impedance is equal to

$$P_r(t) = v_o(t)v_o^*(t) \tag{97}$$

and the expected value

$$P_r = E\{v_o(t)v_o^*(t)\}$$

$$= R_{v_o}(0) \tag{98}$$

where $R_{v_o}(\tau)$ is the autocorrelation of $v_o(t)$. This is valid when assuming stationary processes throughout the whole system. One could represent the mean power in terms of the power spectral density function as

$$P_r = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{v_o}(\omega)d\omega \tag{99}$$

where $S_{v_o}(\omega)$ is the Fourier transform of $R_{v_o}(\tau)$.

It is desired to represent $S_{v_o}(\omega)$ in terms of the spectral density functions of the different inputs to the system, such that one could evaluate the mean power when the spectral density functions of the inputs are given. As shown in Appendix B, Equation (B-42), it is easy to evaluate the spectral density function of an output to a transfer system as a function of the input. Thus, for the system model

$$S_{v_o}(\omega) = S_{v_r}(\omega)\,|G_r(\omega)|^2 \tag{100}$$

and

$$S_{v_i}(\omega) = S_{v_t}(\omega)\,|G_t(\omega)|^2 \tag{101}$$

However, there is no immediate way to evaluate the spectral density function of a multiplication of two processes, especially when they are vectors. The spectral density function of $v_r(t)$ will be derived as follows. Due to Equation (A-41)

$$v_r(t) = \bar{h}_r(t)\cdot\bar{E}_o(t)$$

$$= \bar{h}_r(t)\cdot[E_t(t)*g_s(t)]$$

$$= \bar{h}_r(t)\cdot[(v_i(t)\bar{h}_t(t))*g_s(t)] \tag{102}$$

According to Equation (96)

$$v_r(t) = \bar{h}_r(t)\cdot\bar{h}_t(t-R/c)v_i(t-R/c)Q(R) \tag{103}$$

The effective heights have been modeled in Chapter II as

$$\bar{h}_t(t) = A_1(t)\bar{h}_{t1} + A_2(t)\bar{h}_{t2} \tag{104}$$

and

$$\bar{h}_r(t) = B_1(t)\bar{h}_{r1} + B_2(t)\bar{h}_{r2} \tag{105}$$

where

$$A_1(t) = |A_1(t)|e^{j\alpha_1(t)} \tag{106}$$

$$A_2(t) = |A_2(t)|e^{j\alpha_2(t)} \tag{107}$$

$$B_1(t) = |B_1(t)|e^{-j\beta_1(t)} \tag{108}$$

$$B_2(t) = |B_2(t)|e^{-j\beta_2(t)} \tag{109}$$

The minus signs in front of the β_i's are due to the opposite directions of the receiving and transmitting paths. Thus

$$v_r(t) = Q(R)v_i(t-R/c)[A_1(t-R/c)\bar{h}_{t1}+A_2(t-R/c)\bar{h}_{t2}]\cdot[B_1(t)\bar{h}_{r1}+B_2(t)\bar{h}_{r2}] \tag{110}$$

and

$$v_r^*(t-\tau) = Q^*(R)v_i^*(t-\tau-R/c)[A_1^*(t-\tau-R/c)\bar{h}_{t1}^*+A_2^*(t-\tau-R/c)\bar{h}_{t2}^*]$$
$$\cdot[B_1^*(t-\tau)\bar{h}_{r1}^*+B_2^*(t-\tau)\bar{h}_{r2}^*] \tag{111}$$

The resultant autocorrelation is

$$\begin{aligned}
R_{v_r}(\tau) &= E\{v_r(t)v_r^*(t-\tau)\} \\
&= E\{\bar{h}_r(t)\cdot\bar{h}_t(t-R/c)v_i(t-R/c)Q(R) \\
&\quad \bar{h}_r^*(t-\tau)\cdot\bar{h}_t^*(t-\tau-R/c)v_i^*(t-\tau-R/c)Q^*(R)\} \\
&= |Q(R)|^2\, E\{\bar{h}_r(t)\cdot\bar{h}_t(t-R/c)\bar{h}_r^*(t-\tau)\cdot\bar{h}_t^*(t-\tau-R/c) \\
&\quad v_i(t-R/c)v_i^*(t-\tau-R/c)\}
\end{aligned} \tag{112}$$

It is reasonable to assume that $v_i(t)$ is statistically independent from $\overline{h}_t(t)$ and $\overline{h}_r(t)$. Thus

$$R_{v_r}(\tau) = |Q(R)|^2 R_{v_i}(\tau)\ E\{\overline{h}_r(t)\cdot\overline{h}_t(t-R/c)\overline{h}_r^{\,*}(t-\tau)\cdot\overline{h}_t^{\,*}(t-\tau-R/c)\} \tag{113}$$

where $\overline{h}_{ti}$ and $\overline{h}_{ri}$, $i=1,2$, are assumed to be constant in frequency for $\omega \ll \omega_o$ and to be random variables only. The quantities A_i and B_i, $i=1,2$, are assumed to be random processes in the most general case. As a result, the $\overline{h}$'s and the A's or B's are statistically independent. Therefore

$$\begin{aligned}
R_{v_r}(\tau) &= |Q(R)|^2 R_{v_i}(\tau)\ E\{\overline{h}_r(t)\cdot\overline{h}_t(t-R/c)\overline{h}_r^{\,*}(t-\tau)\cdot\overline{h}_t^{\,*}(t-\tau-R/c)\} \\
&= |Q(R)|^2 R_{v_i}(\tau)\ E\{[A_1(t-R/c)\overline{h}_{t1}+A_2(t-R/c)\overline{h}_{t2}]\cdot[B_1(t)\overline{h}_{r1} \\
&\quad + B_2(t)\overline{h}_{r2}][A_1^*(t-\tau-R/c)\overline{h}_{t1}^{\,*}+A_2^*(t-\tau-R/c)\overline{h}_{t2}^{\,*}]\cdot[B_1^*(t-\tau)\overline{h}_{r1}^{\,*} \\
&\quad + B_2^*(t-\tau)\overline{h}_{r2}^{\,*}]\}
\end{aligned} \tag{114}$$

or

$$\begin{aligned}
R_{v_r}(\tau) &= |Q(R)|^2 R_{v_i}(\tau) \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{l=1}^{2} \\
&\quad E\{A_i(t-R/c)B_j(t)A_k^*(t-\tau-R/c)B_l^*(t-\tau)(\overline{h}_{ti}\cdot\overline{h}_{rj})(\overline{h}_{tk}\cdot\overline{h}_{rl})^*\} \\
&= |Q(R)|^2 R_{v_i}(\tau) \sum_{i,j,k,l=1}^{2} R_{ik}^{jl}(\tau)\ E\{(\overline{h}_{ti}\cdot\overline{h}_{rj})(\overline{h}_{tk}\cdot\overline{h}_{rl})^*\}
\end{aligned} \tag{115}$$

where

$$R_{ik}^{jl}(\tau) = E\{A_i(t-R/c)A_k^*(t-\tau-R/c)B_j(t)B_l^*(t-\tau)\} \tag{116}$$

It is convenient to define a cross-covariance term such as

$$C_{ik}^{jl}(\tau) = E\{[A_i(t-R/c)A_k^*(t-\tau-R/c) - R_{tik}(\tau)][B_j(t)B_l^*(t-\tau) - R_{rjl}(\tau)]\} \tag{117}$$

where

$$R_{tik}(\tau) = E\{A_i(t-R/c)A_k^*(t-\tau-R/c)\} \tag{118}$$

and

$$R_{rjl}(\tau) = E\{B_j(t)B_l^*(t-\tau)\} \tag{119}$$

By this definition, $R_{jk}^{jl}(\tau)$ can be represented as a function of the correlation functions of the receiver and transmitter, $R_{rjl}(\tau)$ and $R_{tik}(\tau)$, respectively. Thus

$$R_{ik}^{jl}(\tau) = R_{tik}(\tau)R_{rjl}(\tau) + C_{ik}^{jl}(\tau) \tag{120}$$

and

$$R_{v_r}(\tau) = |Q(R)|^2 R_{v_i}(\tau) \sum_{i,j,k,l=1}^{2} [R_{tik}(\tau)R_{rjl}(\tau) + C_{ik}^{jl}(\tau)]\ E\{(\bar{h}_{ti}\cdot\bar{h}_{rj})(\bar{h}_{tk}\cdot\bar{h}_{rl})^*\} \tag{121}$$

or

$$R_{v_r}(\tau) = |Q(R)|^2 R_{v_i}(\tau)[R_{\bar{h}_t \cdot \bar{h}_r}(\tau) + C_{\bar{h}_t \cdot \bar{h}_r}(\tau)] \tag{122}$$

where

$$R_{\bar{h}_t \cdot \bar{h}_r}(\tau) = \sum_{i,j,k,l=1}^{2} R_{tik}(\tau) R_{rjl}(\tau) \; E\{(\bar{h}_{ti} \cdot \bar{h}_{rj})(\bar{h}_{tk} \cdot \bar{h}_{rl})^*\} \tag{123}$$

and

$$C_{\bar{h}_t \cdot \bar{h}_r}(\tau) = \sum_{i,j,k,l=1}^{2} C_{ik}^{jl}(\tau) \; E\{(\bar{h}_{ti} \cdot \bar{h}_{rj})(\bar{h}_{tk} \cdot \bar{h}_{rl})^*\} \tag{124}$$

When transforming into frequency space,

$$S_{v_r}(\omega) = |Q(R)|^2 S_{v_i}(\omega) * S_{\bar{h}_t \cdot \bar{h}_r}(\omega) \tag{125}$$

where

$$S_{v_r}(\omega) = \mathfrak{F}[R_{v_r}(\tau)] \tag{126}$$

$$S_{v_i}(\omega) = \mathfrak{F}[R_{v_i}(\tau)] \tag{127}$$

and

$$S_{\bar{h}_t \cdot \bar{h}_r}(\omega) = \mathfrak{F}\,[R_{\bar{h}_t \cdot \bar{h}_r}(\tau) + C_{\bar{h}_t \cdot \bar{h}_r}(\tau)]$$

$$= \mathfrak{F}\,[R_{\bar{h}_t \cdot \bar{h}_r}(\tau)] + \mathfrak{F}\,[C_{\bar{h}_t \cdot \bar{h}_r}(\tau)]$$

$$= S_R(\omega) + S_c(\omega) \tag{128}$$

Substituting Equation (101), Equation (125) becomes

$$S_{v_r}(\omega) \simeq |Q(R)|^2[S_{v_t}(\omega)|G_t(\omega)|^2]*S_{\bar{h}_t \cdot \bar{h}_r}(\omega) \tag{129}$$

and

$$P_r = \frac{|Q(R)|^2}{2\pi}\int_{-\infty}^{\infty} |G_r(\omega)|^2\{[S_{v_t}(\omega)|G_t(\omega)|^2]*S_{\bar{h}_t \cdot \bar{h}_r}(\omega)\}d\omega \tag{130}$$

One can break down the correlation function of the dot product $\bar{h}_t \cdot \bar{h}_r$ by using Equation (B-27). Thus,

$$R_{tik}(\tau) = E\{A_i(t-R/c)A_k^*(t-\tau-R/c)\}$$

$$= \eta_{ti}\eta_{tk}^* + C_{tik}(\tau) \tag{131}$$

and

$$R_{rjl}(\tau) = E\{B_j(t)B_l^*(t-\tau)\}$$

$$= \eta_{rj}\eta_{rl}^* + C_{rjl}(\tau) \tag{132}$$

where

$$\eta_{ti} = E\{A_i(t-R/c)\} \tag{133}$$

$$\eta^*_{tk} = E\{A^*_k(t-\tau-R/c)\} \tag{134}$$

$$\eta_{rj} = E\{B_j(\tau)\} \tag{135}$$

and

$$\eta^*_{rl} = E\{B^*_l(t-\tau)\} \tag{136}$$

Therefore

$$\begin{aligned} R^{jl}_{ik}(\tau) &= R_{tik}(\tau)R_{rjl}(\tau) + C^{jl}_{ik}(\tau) \\ &= [\eta_{ti}\eta^*_{tk} + C_{tik}(\tau)][\eta_{rj}\eta^*_{rl} + C_{rjl}(\tau)] + C^{jl}_{ik}(\tau) \\ &= \eta_{ti}\eta_{rj}\eta^*_{tk}\eta^*_{rl} + \eta_{ti}\eta^*_{tk}\, C_{rjl}(\tau) + \eta_{rj}\eta^*_{rl}\, C_{tik}(\tau) \\ &\quad + C_{tik}(\tau)C_{rjl}(\tau) + C^{jl}_{ik}(\tau) \end{aligned} \tag{137}$$

where $C_{tik}(\tau)$ and $C_{rjl}(\tau)$ are the cross-covariance of transmitter and receiver, respectively.

In summary,

$$P_r = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{v_o}(\omega)d\omega \tag{138}$$

where

$$S_{v_o}(\omega) = |G_r(\omega)|^2 |G_s|^2 \{S_{\bar{h}_t \cdot \bar{h}_r}(\omega) * [|G_t(\omega)|^2 S_{v_t}(\omega)]\} \qquad (139)$$

$$S_{\bar{h}_t \cdot \bar{h}_r}(\omega) = \sum_{ijkl} [R_{ik}^{jl}(\tau)] \; E\{(\bar{h}_{ti} \cdot \bar{h}_{rj})(\bar{h}_{tk} \cdot \bar{h}_{rl})^*\} \qquad (140)$$

and

$$R_{ik}^{jl}(\tau) = E\{A_i(t-R/c)A_k^*(t-\tau-R/c)B_j(t)B_l^*(t-\tau)\} \qquad (141)$$

In a more convenient way,

$$S_{\bar{h}_t \cdot \bar{h}_r}(\omega) = S_R(\omega) + S_c(\omega) \qquad (142)$$

where

$$S_R(\omega) = \sum_{ijkl} S_{tik}(\omega) * S_{rjl}(\omega) \; E\{(\bar{h}_{ti} \cdot \bar{h}_{rj})(\bar{h}_{tk} \cdot \bar{h}_{rl})^*\} \qquad (143)$$

with

$$\begin{aligned} S_{tik}(\omega) &= [R_{tik}(\tau)] \\ &= [E\{A_i(t-R/c)A_k^*(t-\tau-R/c)\}] \end{aligned} \qquad (144)$$

$$\begin{aligned} S_{rjl}(\omega) &= [R_{rjl}(\tau)] \\ &= [E\{B_j(t)B_l^*(t-\tau)\}] \end{aligned} \qquad (145)$$

and

$$S_c(\omega) = \sum_{ijkl} [C_{ik}^{jl}(\tau)]\ E\{(\bar{h}_{ti}\cdot\bar{h}_{rj})(\bar{h}_{tk}\cdot\bar{h}_{rl})^*\} \tag{146}$$

with

$$C_{ik}^{jl}(\tau) = E\{[A_i(t-R/c)A_k^*(t-\tau-R/c) - R_{tik}(\tau)][B_j(t)B_l^*(t-\tau) - R_{rjl}(\tau)]\} \tag{147}$$

Thus, the averaged received power of a receiving system, when polarization diversity processes are employed by the receiver and the transmitter, depends upon the spectral characteristic of the correlation between the two processes. This spectrum is spread in a convolution manner by the transmitter frequency spectrum. However, this convolved spectrum will contribute to the average power only within the bandwidth of the receiving system. The parts of the spectrum which are outside of the receiver bandwidth are wasted.

Assumptions

Equation (138) can be simplified by assuming the following assumptions:

(1) The two processes, the transmitting and the receiving, are statistically independent.

(2) The two polarization components are statistically independent for each antenna system, both transmitting and receiving.

(3) Only one of the systems is a random process and the other is a random variable.

(4) A typical transmitter where $v_t(t)$ = constant and a typical receiver where $G_r(\omega)$ is an ideal low pass filter around the carrier frequency.

(5) $S_{v_t}(\omega)$ is a narrow band noise.

Assumption (1) yields

$$S_c(\omega) = 0 \tag{148}$$

and

$$S_{\overline{h}_t \cdot \overline{h}_r}(\omega) = S_R(\omega)$$

$$= \sum_{i,j,k,l=1}^{2} S_{tik}(\omega) * S_{rjl}(\omega) \; E\{(\overline{h}_{ti} \cdot \overline{h}_{rj})(\overline{h}_{tk} \cdot \overline{h}_{rl})^*\} \tag{149}$$

where

$$S_{tik}(\omega) = \quad [E\{A_i(t-R/c)A_k^*(t-\tau-R/c)\}] \tag{150}$$

and

$$S_{rjl}(\omega) = \quad [E\{B_j(\tau)B_l^*(t-\tau)\}] \tag{151}$$

Since $\overline{h}_t$ and $\overline{h}_r$ are statistically independent the expected value of the dot product of their components can be simplified. According to the model which has been used, the components of each system, the transmitter and the receiver, are orthogonal. Thus, when using the directions of the

components of one of the systems, the transmitter for example, one obtains

$$E\{(\bar{h}_{ti}\cdot\bar{h}_{rj})(\bar{h}_{tk}\cdot\bar{h}_{rl})^*\} = E\{h_{ti}h_{rj_{ti}}\ h^*_{tk}h^*_{rl_{tk}}\}$$

$$= E\{h_{ti}h^*_{tk}\}\ E\{h_{rj_{ti}}\ h^*_{rl_{tk}}\} \tag{152}$$

where

$$h_{ti} = \bar{h}_{ti}\cdot\hat{ti}^* \tag{153}$$

$$h_{rj_{ti}} = \bar{h}_{rj}\cdot\hat{ti} \tag{154}$$

$\hat{ti}$ is the direction of $\bar{h}_{ti}$, and the other terms are obtained similarly. Thus,

$$S_{\bar{h}_t\cdot\bar{h}_r}(\omega) = \sum_{i,j,k,l=1}^{2} S_{tik}(\omega)*S_{rjl}(\omega)\ E\{h_{ti}h^*_{tk}\}\,E\{h_{rj_{ti}}\ h^*_{rl_{tk}}\} \tag{155}$$

Due to Assumption (2), Equations (131) and (132) can be simplified, as follows

$$R_{tik}(\tau) = \eta_{ti}\eta^*_{tk} + C_{tik}(\tau)$$

$$= \begin{cases} |\eta_{ti}|^2 + C_{ti}(\tau) & , \quad i=k \\ \eta_{ti}\eta^*_{tk} & , \quad i\neq k \end{cases} \tag{156}$$

where $C_{ti}(\tau)$ is the autocovariance of $A_i(t)$, and

$$R_{rjl}(\tau) = \eta_{rj}\eta^*_{rl} + C_{rjl}(\tau)$$

$$= \begin{cases} |\eta_{rj}|^2 + C_{rj}(\tau) & , \ j=l \\ \eta_{rj}\eta^*_{rl} & , \ j\neq l \end{cases} \tag{157}$$

where $C_{rj}(\tau)$ is the autocovariance of $B_j(t)$. Equation (152) can be further simplified when assuming (2). Thus,

$$E\{h_{ti}h^*_{tk}\} = \eta_{h_{ti}}\eta^*_{h_{tk}} + \mu_{tik}$$

$$= \begin{cases} |\eta_{h_{ti}}|^2 + \mu_{ti} & , \ i=k \\ \eta_{h_{ti}}\eta^*_{h_{tk}} & , \ i\neq k \end{cases} \tag{158}$$

and

$$E\{h_{rj_{ti}} h^*_{rl_{tk}}\} = \eta_{h_{rj_{ti}}} \eta^*_{h_{rl_{tk}}} + \mu_{rjl_{tik}}$$

$$= \begin{cases} \eta_{h_{rj_{ti}}} \eta^*_{h_{rj_{tk}}} + \mu_{rj_{tik}} & , \ j=l \\ \eta_{h_{rj_{ti}}} \eta^*_{h_{rl_{tk}}} & , \ j\neq l \end{cases} \tag{159}$$

where μ_{tik} is the covariance of h_{ti} and h_{tk}, and $\mu_{rjl_{tik}}$ is the covariance of components of h_{rj} and h_{rl} in $\hat{ti}$ and $\hat{tk}$ directions respectively.

Assumption (3) resolves the convolution form of Equation (149) into a function of the spectral density function of the system which employs a random process while the other system does not contribute to the spectral characteristics of the received power, except for a constant. Since one of the systems behaves as a random variable, there is no statistical dependence between the two systems and the results of Assumption (1) can be used in this case. Assuming the receiving system to be a random variable yields the spectral behavior

$$S_{\overline{h}_t \cdot \overline{h}_r}(\omega) = \sum_{i,j,k,l=1}^{2} S_{tik}(\omega)\, E\{B_j B_l^*\} E\{h_{ti} h_{tk}^*\} E\{h_{rj_{ti}} h^*_{rl_{tk}}\} \tag{160}$$

Assumption (4) allows an explicit insight into the spectral characteristics of the received power. This insight indicates the capability of an optimization technique which is discussed in detail in Chapter IV. Since $v_t(t)$ = constant,

$$\begin{aligned} R_{v_t}(\tau) &= E\{v_t(t) v_t^*(t-\tau)\} \\ &= \text{constant} \end{aligned} \tag{161}$$

and

$$S_{v_t}(\omega) = D\delta(\omega) \tag{162}$$

then S_{v_i} may be written as

$$S_{v_i}(\omega) = D\delta(\omega) |G_t(\omega)|^2 \tag{163}$$

With the receiver definition

$$
|G_r(\omega)|^2 = \begin{cases} |G_r|^2 & , \quad |\omega| \leq \omega_c \\ 0 & , \quad |\omega| > \omega_c \end{cases} \tag{164}
$$

one can resolve Equation (130) to obtain

$$
P_r = \frac{D}{2\pi} |Q(R)|^2 |G_r|^2 |G_t(0)|^2 \int_{-\omega_c}^{\omega_c} S_{\overline{h}_t \cdot \overline{h}_r}(\omega) d\omega \tag{165}
$$

since

$$
S_{\overline{h}_t \cdot \overline{h}_r}(\omega) * [|G_t(\omega)|^2 \delta(\omega)] = \int_{-\infty}^{\infty} S_{\overline{h}_t \cdot \overline{h}_r}(\omega - \omega') |G_t(\omega')|^2 \delta(\omega') d\omega'
$$

$$
= S_{\overline{h}_t \cdot \overline{h}_r}(\omega) |G_t(0)|^2 \tag{166}
$$

The results of Assumption (4) are changed only slightly with Assumption (5). The only difference is that the spectral density function $S_{\overline{h}_t \cdot \overline{h}_r}(\omega)$ is spread, in a convolution manner, by the spectrum of the noise of the transmitter.

IV. Statement of Problems

One should consider two opposite problems concerning polarization diversity. Both problems are dealing with the question of how to optimize the likelihood of reception of a receiving system when a polarization diversity phenomenon is employed by both the receiving and the transmitting systems. The first problem is how to maximize the likelihood of reception of the receiving system in order to effectively disrupt reception. This problem is referred to as the Electromagnetic Counter Measure (ECM) problem. The second problem is how to minimize the likelihood of reception of interference by the receiving system in order to avoid jamming. This problem is referred to as the Electromagnetic Counter-Counter Measure (ECCM) problem. In both cases, there are two opposing systems, the receiver and the jammer, where one system is assumed to have some kind of polarization diversity technique and the designer has to find an optimized polarization diversity technique for the opposing system such that the likelihood of reception is optimized in the desired sense. The "assumed" polarization diversity technique can employ a random process in the most general case, or it can be simply a random variable, meaning that the polarization can take on different values with each value having some probability of occurrence.

Optimization Factors

One of the objectives of this study is to point out some optimization techniques by employing random processes with the polarization diversity

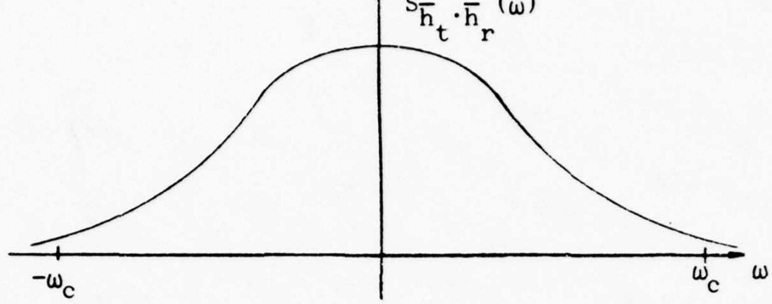

Figure 9. Maximum Spectrum

system model which was developed in Chapter II. One can use the result of the most general case as it appears in Equation (137). However, for the sake of simplicity, several assumptions are made in order to point out an optimized technique of polarization diversity. Some specific limitations on implemetation are also considered. Using Assumption (4) to define a simple system, one obtains the spectral function of the received power as given by Equation (164)

$$P_r = C \int_{-\omega_c}^{\omega_c} S_{\bar{h}_t \cdot \bar{h}_r}(\omega)d\omega \tag{166}$$

where $S_{\bar{h}_t \cdot \bar{h}_r}(\omega)$ is defined by Equation (139).

From the ECM point of view, optimal reception occurs when most of the energy of the spectral density function $S_{\bar{h}_t \cdot \bar{h}_r}(\omega)$ is between $-\omega_c$ and ω_c as illustrated in Figure 9. Optimal reception, from the ECCM

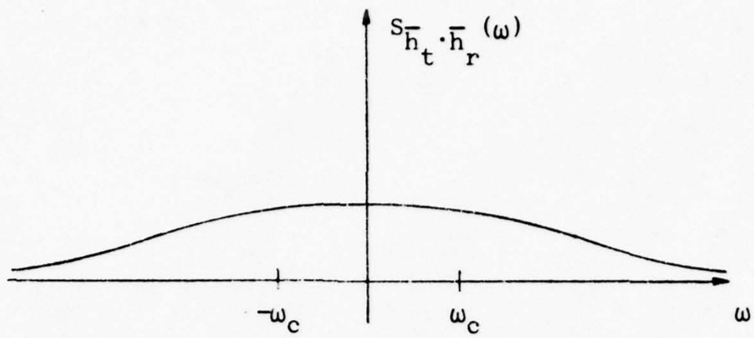

Figure 10. Minimum Spectrum

viewpoint, is achieved by spreading the spectrum of the received power such that most of its energy lies outside the bandpass $-\omega_c$ to ω_c, as illustrated in Figure 10. The behavior of the amplitude of $S_{\bar{h}_t \cdot \bar{h}_r}(\omega)$ is another factor in the optimization procedure. To minimize the likelihood of reception, one may desire the amplitude of $S_{\bar{h}_t \cdot \bar{h}_r}(\omega)$ to approach zero. This is achieved when the two processes, $\bar{h}_t(t)$ and $\bar{h}_r(t)$, are statistically orthogonal. Maximum likelihood of reception occurs when the two processes are statistically dependent. Obviously, it is difficult to implement a complete dependence and one may assume, for simplicity, that the two processes, $\bar{h}_t(t)$ and $\bar{h}_r(t)$, are statistically independent as stated by Assumption (1).

Evaluating Typical Scenarios

Some typical scenarios are evaluated according to the system model which was presented in the last two chapters. Four types of scenarios are considered.

(a) The polarization state of the jammer is fixed while the polarization state of the receiver is assumed to be a random variable.

(b) The configuration is the same as in the first case except that it is considered as an ECCM problem.

(c) The jammer employs a random process in order to overcome the ECCM technique used in the second case.

(d) The receiver employs also a random process in order to reduce the jamming effectiveness.

For evaluating the received power in the first scenario one may go back to the representation of the polarization state by the Poincare sphere, or specifically to Equation (37), where the power is given as a function of the angle between the polarization states, of the jammer and the radar, such that

$$P = E\{\tfrac{1}{2}(1 + \cos\epsilon)\} \tag{167}$$

where P is the normalized average power. Five different cases of scenario (a) are listed in Table I. As seen in the table, in most of the cases, the likelihood of reception is optimized from the jammer point of view. It may be expected that the polarization state of the radar will be adjusted to be orthogonal to the jammer such that the average power is reduced. This status is categorized under scenario (b).

The obvious action that may be taken for improving jamming effectiveness is employing a random process. This case, listed as

Table I

Several Combinations of Scenario (a)

Fixed Polarization	Random Polarization	Probability Density of $\varepsilon = (0,\pi)$	P_{avg}	Nulls
Any	Uniformly distributed on the Poincare sphere	$\frac{1}{2}\sin\varepsilon$	½	Yes (Probability = 0.0)
Linear	Uniformly distributed linear polarization	$\frac{1}{\pi}$	½	Yes (Probability = 0.0)
Circular	Uniformly distributed linear polarization	$\delta(\varepsilon-\pi/2)$	½	No
Linear	Circular Probability of left or right hand is equal to 0.5	$\delta(\varepsilon-\pi/2)$	½	No
Circular	Circular Probability of left or right hand is equal to 0.5	$\frac{1}{2}[\delta(\varepsilon-0^+)+\delta(\varepsilon-\pi^-)]$	½	Yes (Probability = 0.5)

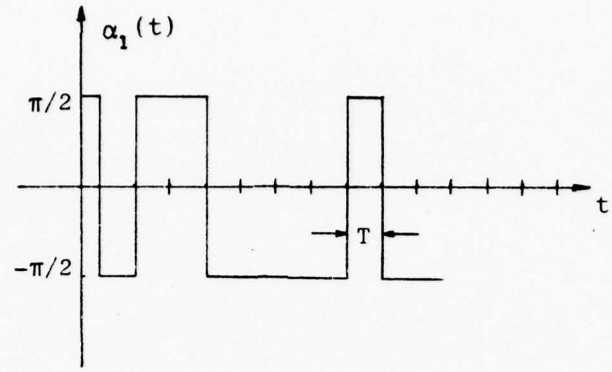

Figure 11. Random Binary Transmission

scenario (c), is evaluated analytically and numerically where both results are presented and compared. The scenario is defined by the four amplitudes of the system model such that

$$
\begin{aligned}
A_1(t) &= e^{j\alpha_1(t)} \\
A_2(t) &= 1 \\
B_1(t) &= 1 \\
B_2(t) &= 0
\end{aligned}
\tag{168}
$$

where

$$\alpha_1(t) = \{-\pi/2, + \pi/2\} \tag{169}$$

and it is a pseudo random process called random binary transmission as illustrated in Figure 11. The B's are selected as such for the ease of evaluation. $\bar{h}_{r1}$ is assumed to be a random variable with equal

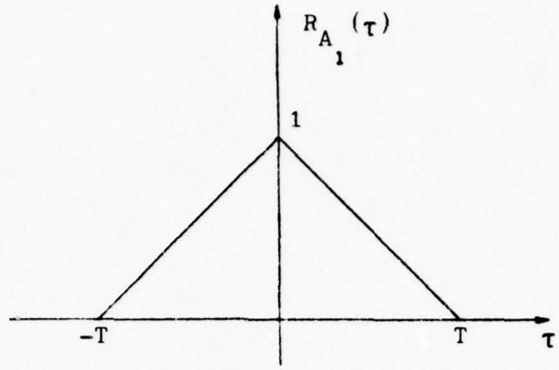

Figure 12. Autocorrelation of Random Binary Transmission

Probability of being oriented in $\hat{\theta}$ or $\hat{\phi}$. $\bar{h}_{t_1}$ and $\bar{h}_{t_2}$ are assumed to be oriented in $\hat{\theta}$ and $\hat{\phi}$, respectively. Thus,

$$R_{ik}^{jl}(\tau) = \tfrac{1}{4}(1 + R_{A_1}(\tau)) \tag{170}$$

where

$$R_{A_1}(\tau) = \begin{cases} 1 - \dfrac{|\tau|}{T} & , |\tau| < T \\ 0 & |\tau| > T \end{cases} \tag{171}$$

as illustrated in Figure 12. The spectral density function of $R_{A_1}(\tau)$, $S_{A_1}(\omega)$, is given as

$$S_{A_1}(\omega) = T \frac{\sin^2(\pi Tf)}{(\pi Tf)^2} \tag{172}$$

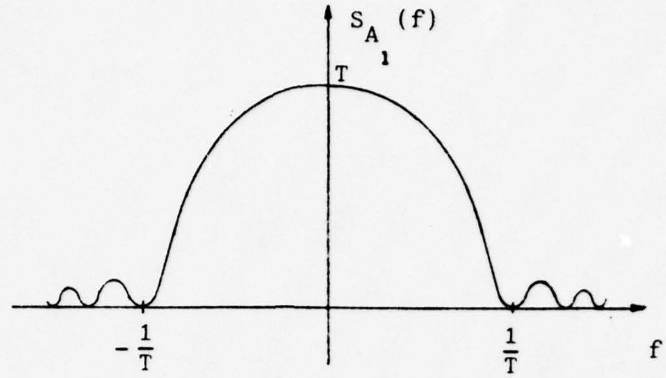

Figure 13. Power Spectral Density of $A_1(t)$

as illustrated in Figure 13. Thus, the average power, P_r, is given a

$$
\begin{aligned}
P_r &= \tfrac{1}{4}\int_{\infty}^{\infty} [\delta(f) + S_{A_1}(f)]df \\
&= \tfrac{1}{2}
\end{aligned}
\tag{173}
$$

assuming the spectrum of the jammer to be within the radar bandwidth. Most of the energy of the spectrum $S_{A_1}(f)$ is in the range $|f| \leq \frac{1}{T}$. Hence, when the radar is band limited, jamming is effective only when

$$T > \frac{1}{f_c} \tag{174}$$

where f_c is the cutoff frequency of the radar bandwidth.

Scenario (c) is also evaluated numerically by the use of SAP3 program found in the AFIT computer library. The random process employed by the jammer is constructed to be similar to the Random Binary Transmission illustrated in Figure 11. The process is based on the sign of

a zero mean, unit variance, Gaussian random noise filtered by several types of low pass filters. Figure 14 illustrated the process, created by a 320KHz filter. Figures 15 and 16 illustrate the time autocorrelation and the spectral density functions of this process. Assuming erogdicity, time and ensemble autocorrelation may be equated which allows comparison of the analytical and numerical analysis. The spectral density function is integrated along the frequency domain, as illustrated in Figure 17. The value of the integral for a certain frequency is equivalent to the expected received power obtained by passing the process through the receiver with a bandwidth equal to that frequency. Figure 18 illustrates the spectral density function constructed by filtering by 640KHz. Figure 19 illustrates the expected received power versus the receiver bandwidth. In Figures 20 and 21 the spectral density function and the received power are illustrated as before except for a process created by a 1.28MHz filter. The filtering of the process has the same influence on jamming effectiveness as the changing of T in the analytical analysis does. Thus, for a certain receiver bandwidth the expected received power decreases when the bandwidth of the random process increases. Thus, as the bandwidth of the polarization diversity process becomes larger than the bandwidth of the radar system, the jamming effectiveness goes down.

The last scenario, (d), is analyzed numerically due to its complexity. The jamming process is assumed to be as before and the radar polarization diversity process is assumed to be similar to the jamming process. The radar process is assumed to be created by a 640KHz filter for all cases. The process of the radar system is employed by the polarization amplitude controller $B_1(t)$. In Figures 22 through 27, the

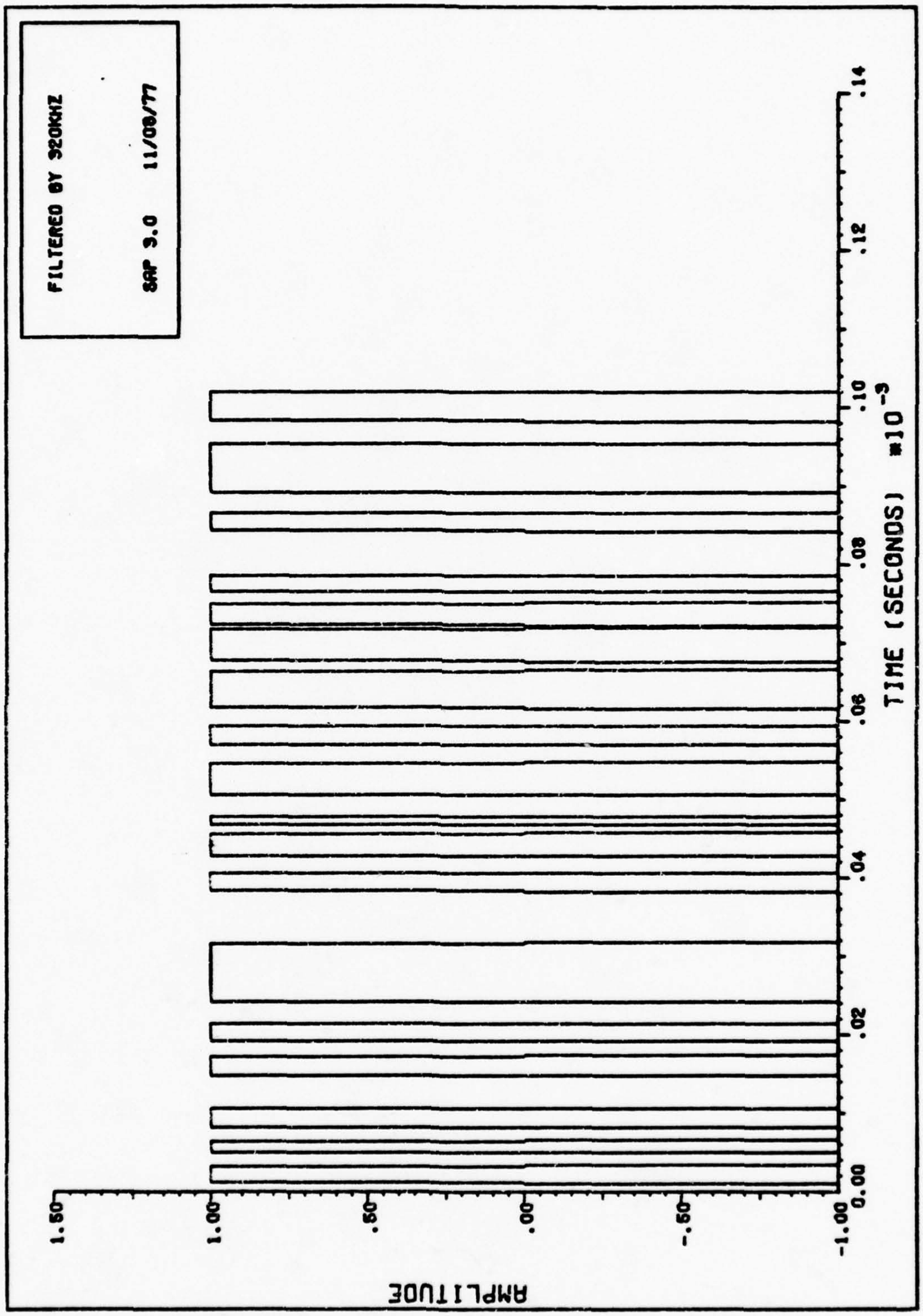

FIGURE 14. RANDOM BINARY TRANSMISSION

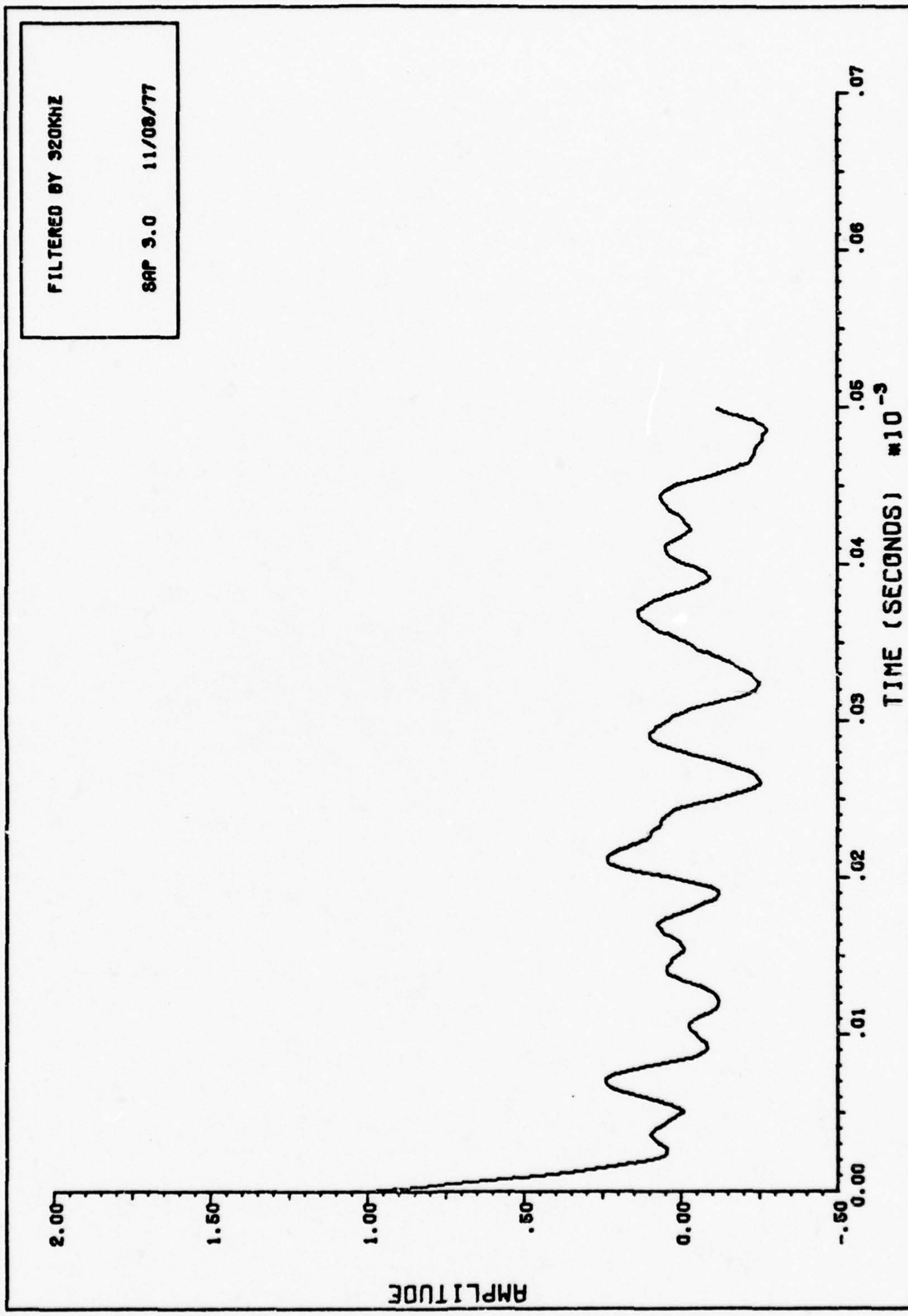

FIGURE 15. AUTOCORRELATION FUNCTION

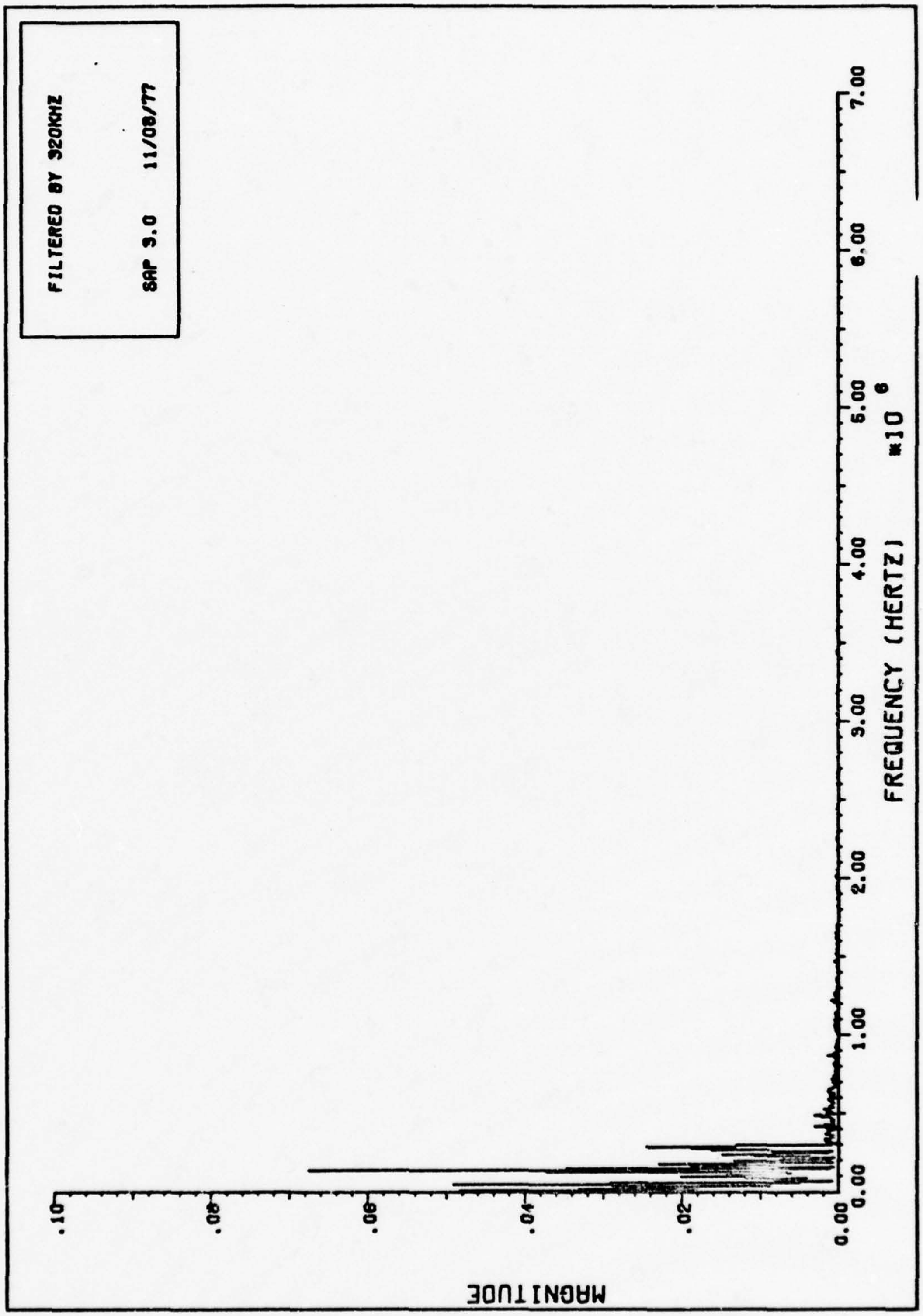

FIGURE 16. SPECTRAL DENSITY FUNCTION

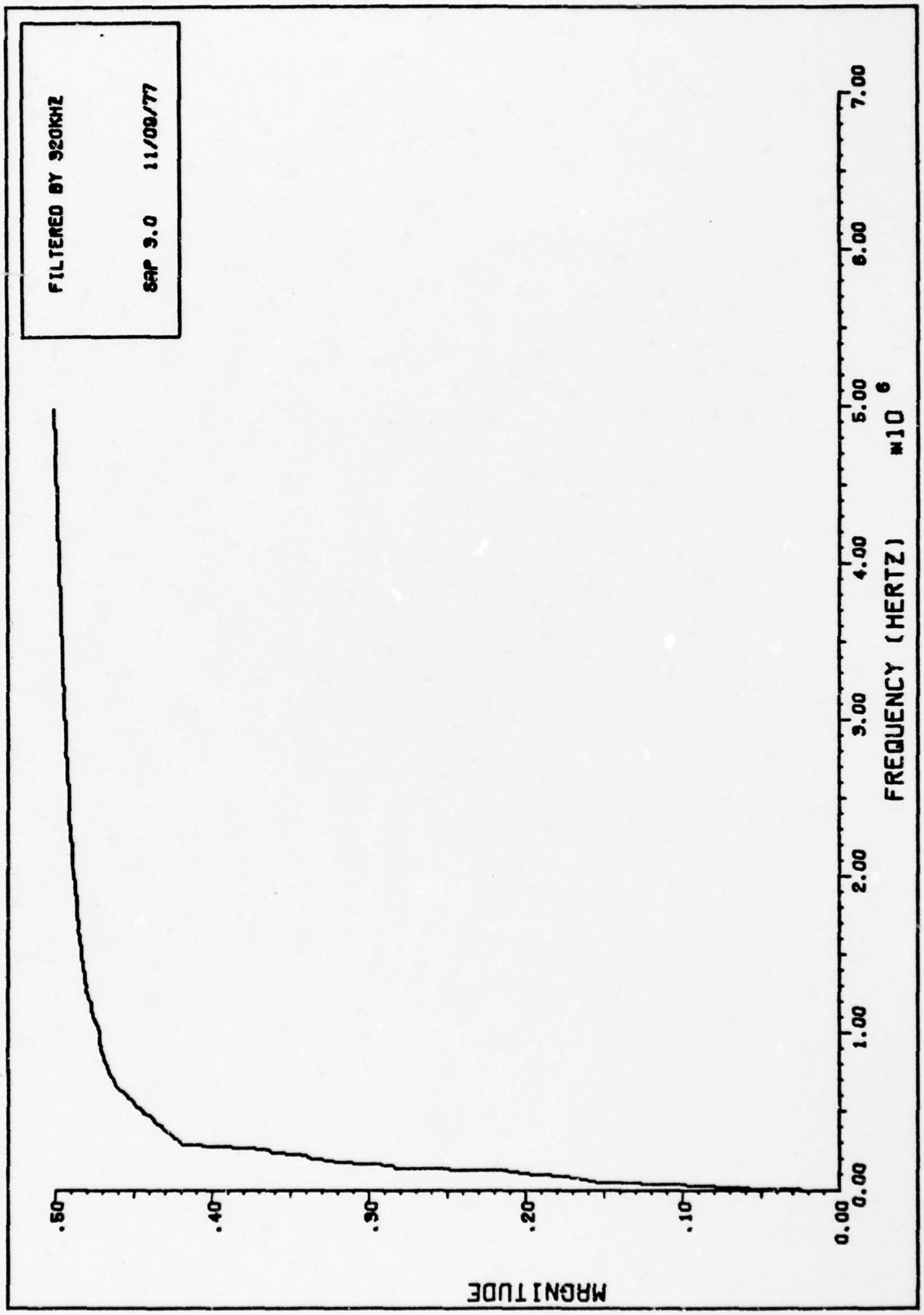

FIGURE 17.RECEIVED POWER VS RECEIVER BANDWIDTH

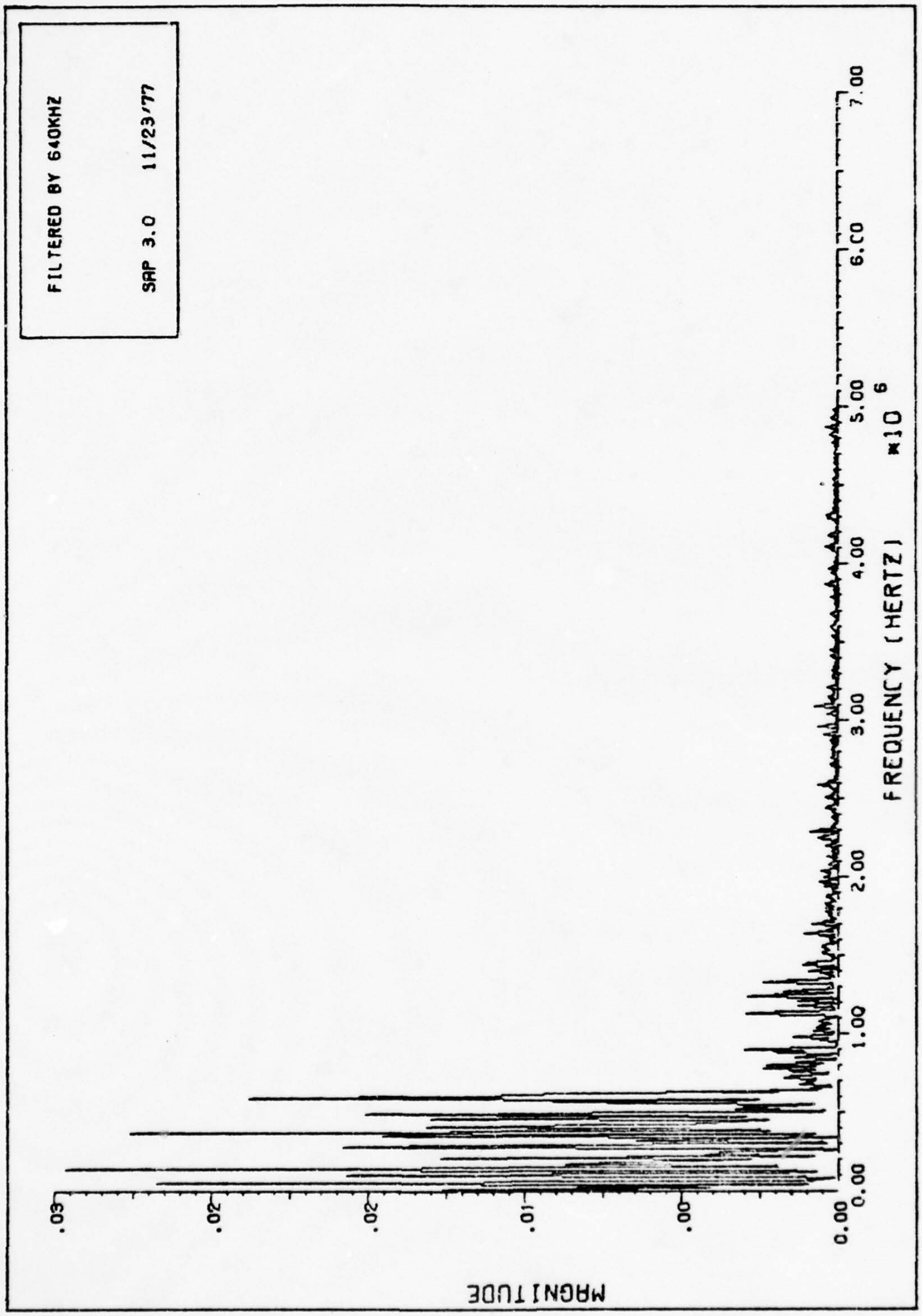

FIGURE 18. SPECTRAL DENSITY FUNCION

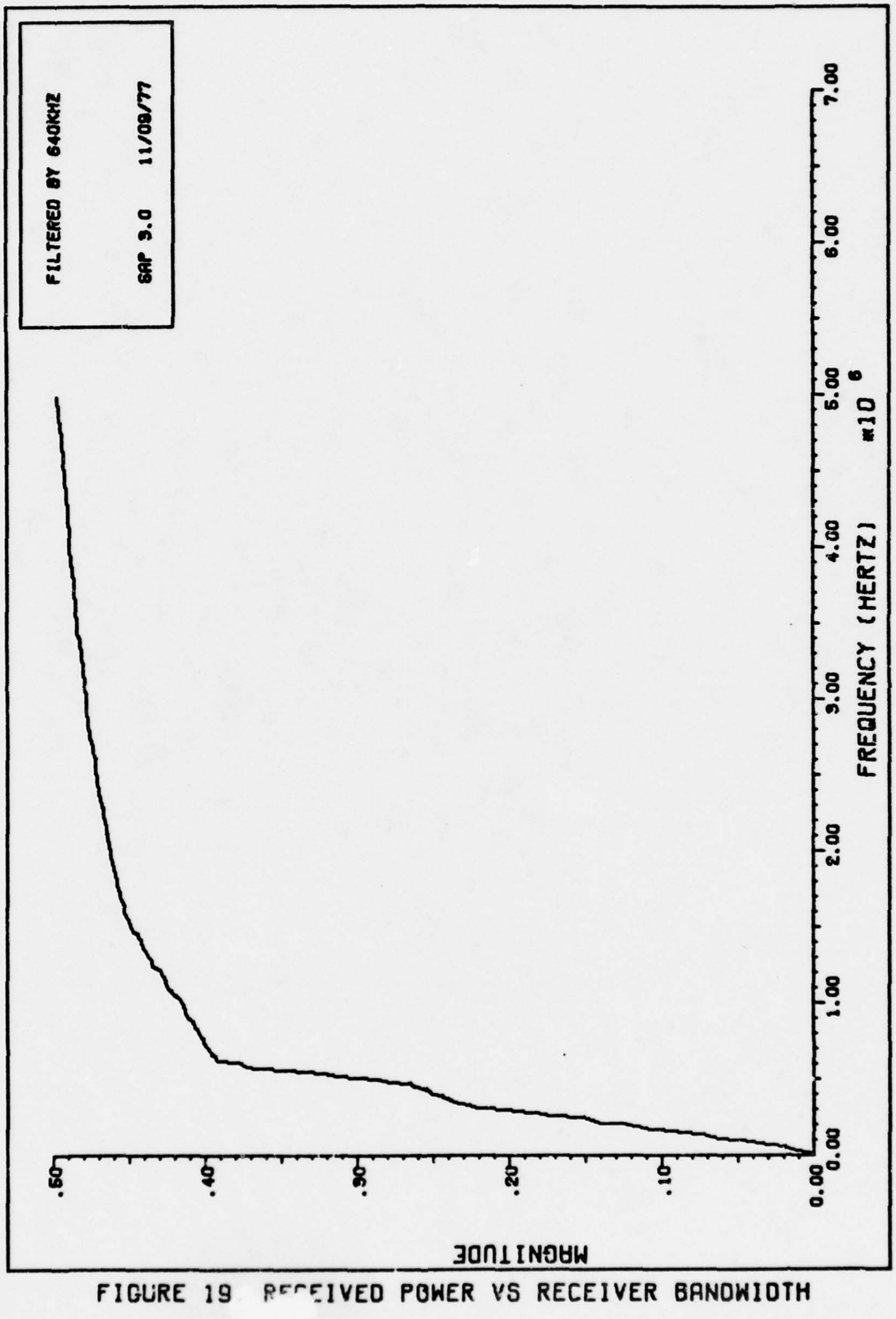

FIGURE 19 RECEIVED POWER VS RECEIVER BANDWIDTH

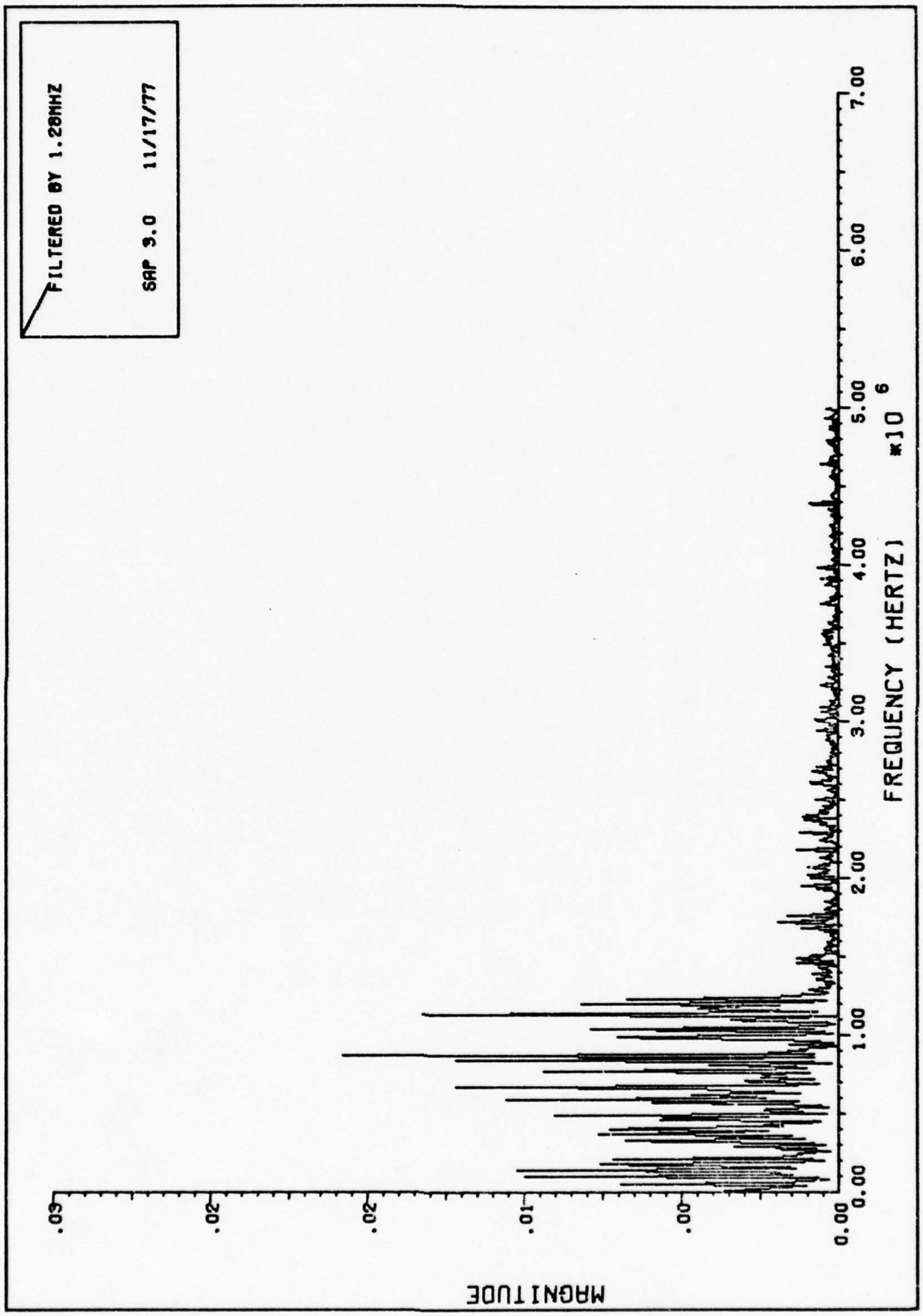

FIGURE 20. SPECTRAL DENSITY FUNCTION

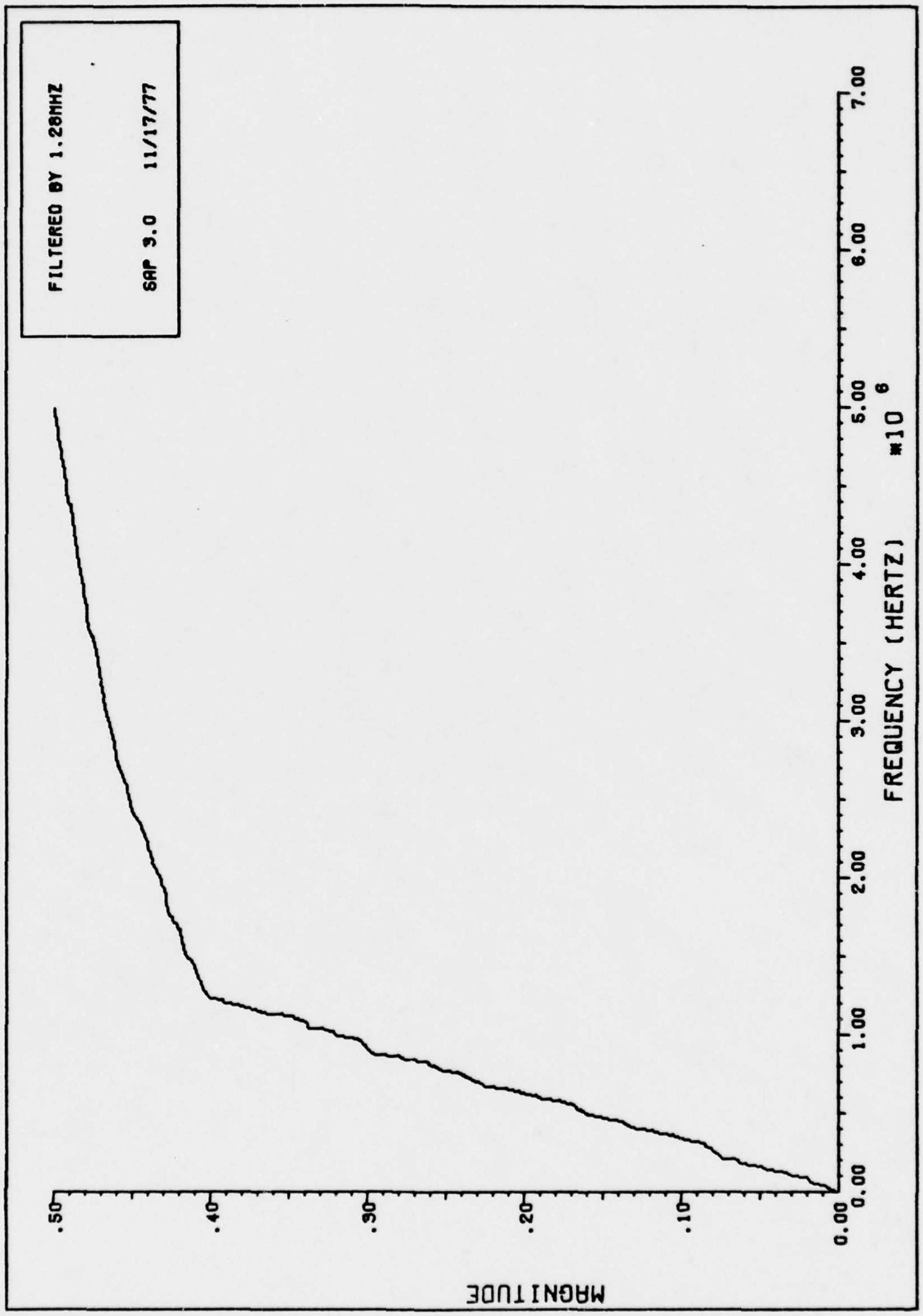

FIGURE 21. RECEIVED POWER VS RECEIVER BANDWIDTH

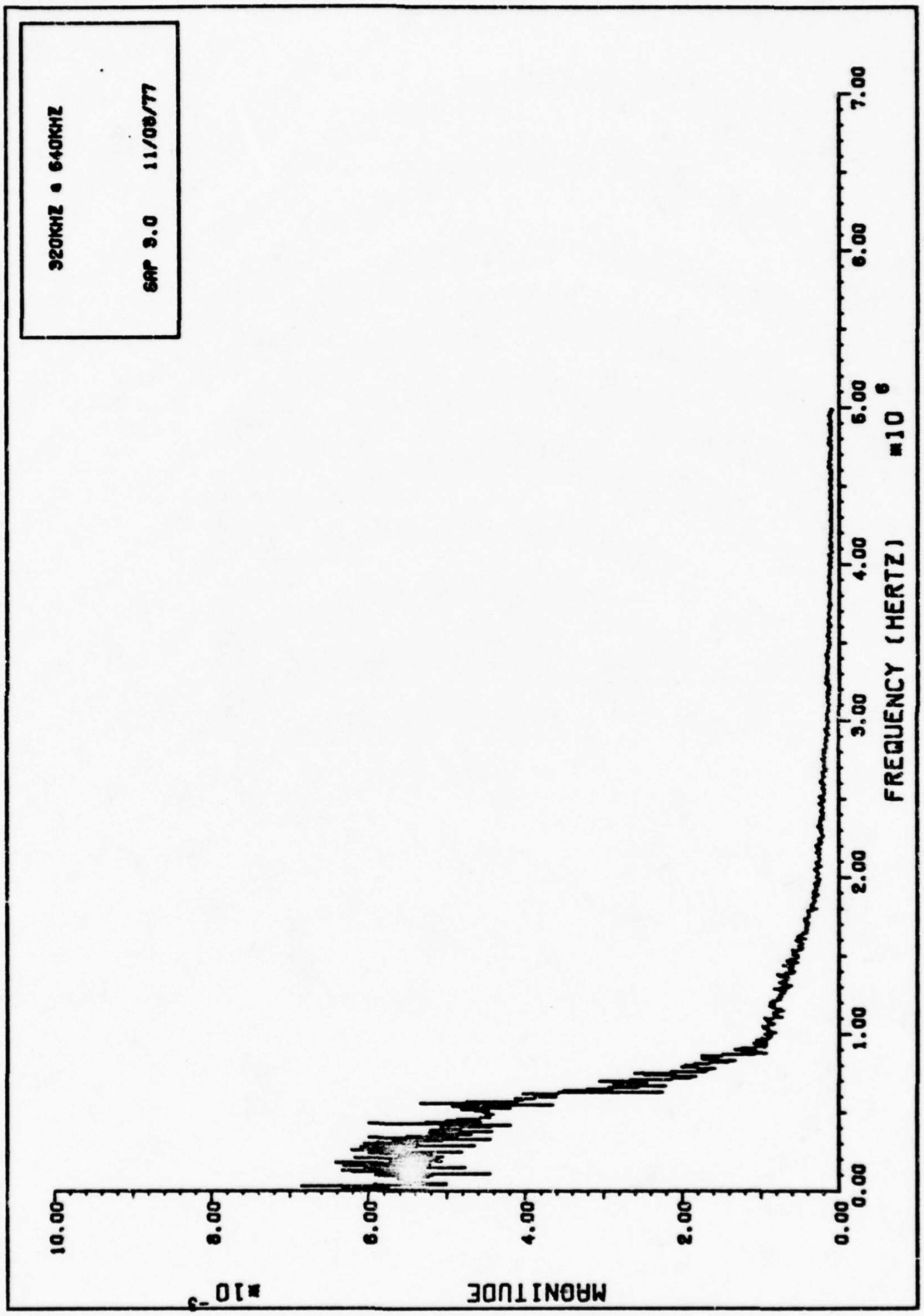

FIGURE 22. JOINT SPECTRUM

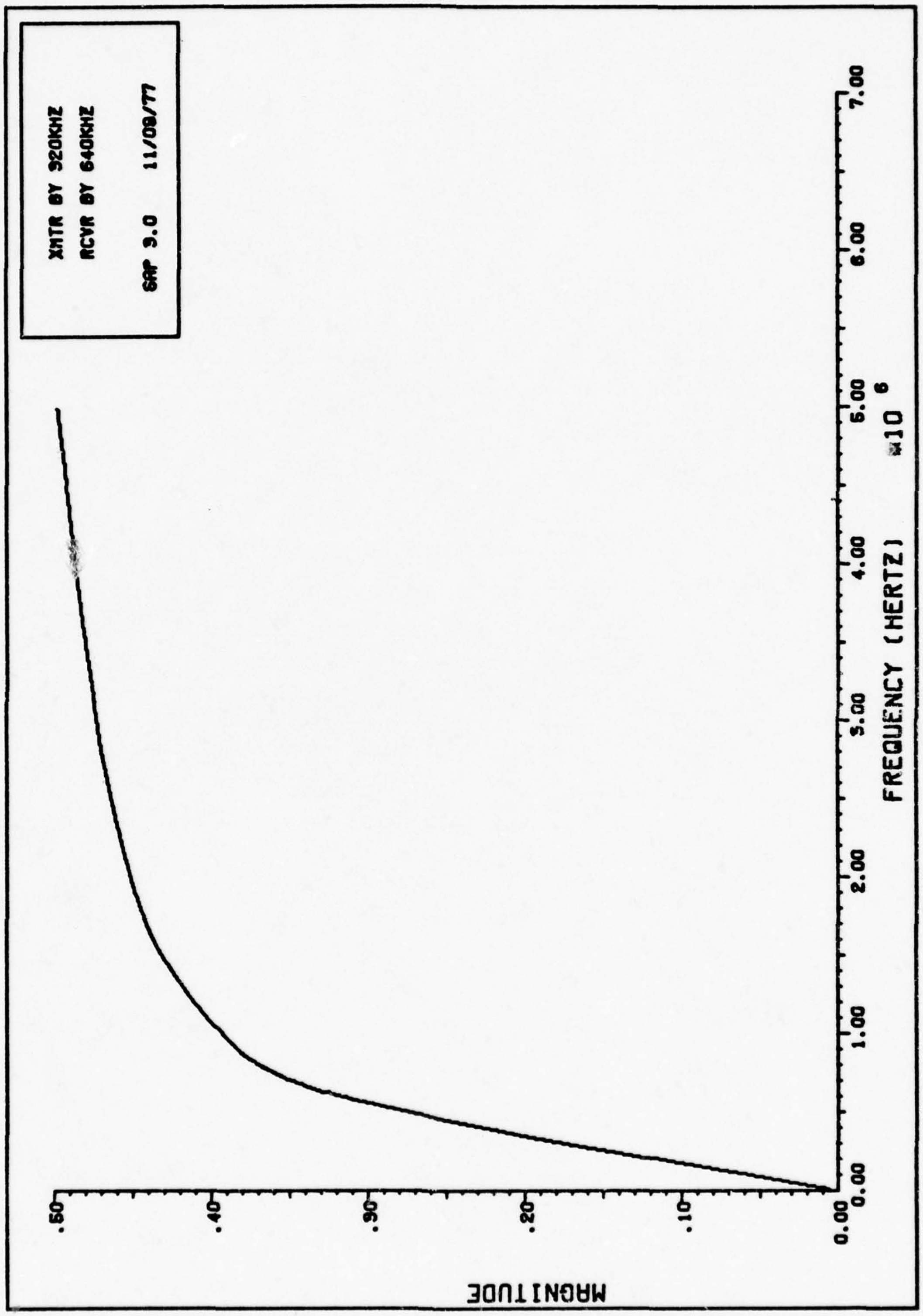

FIGURE 23. RECEIVED POWER VS RECEIVER BANDWIDTH

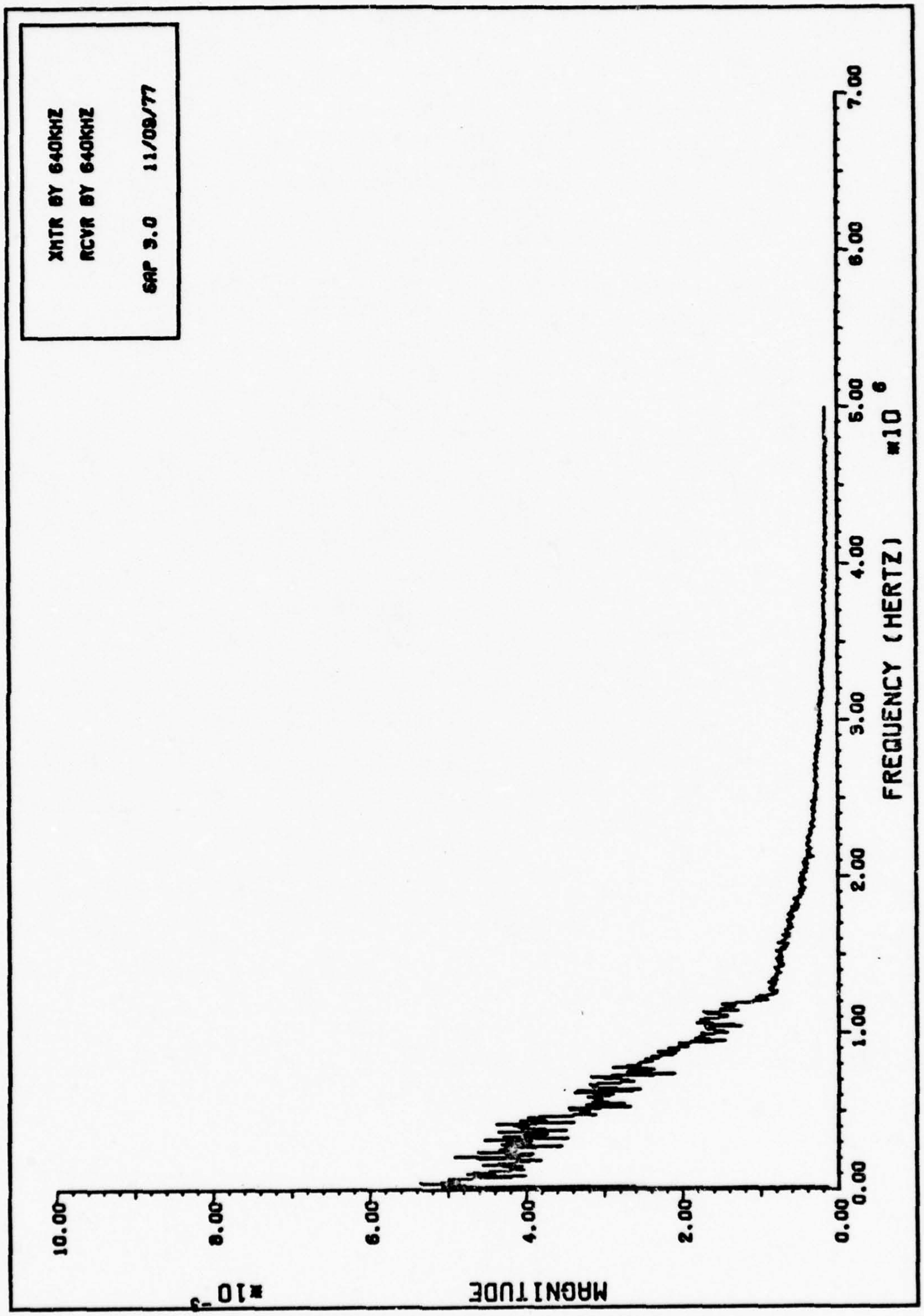

FIGURE 24. JOINT SPECTRUM

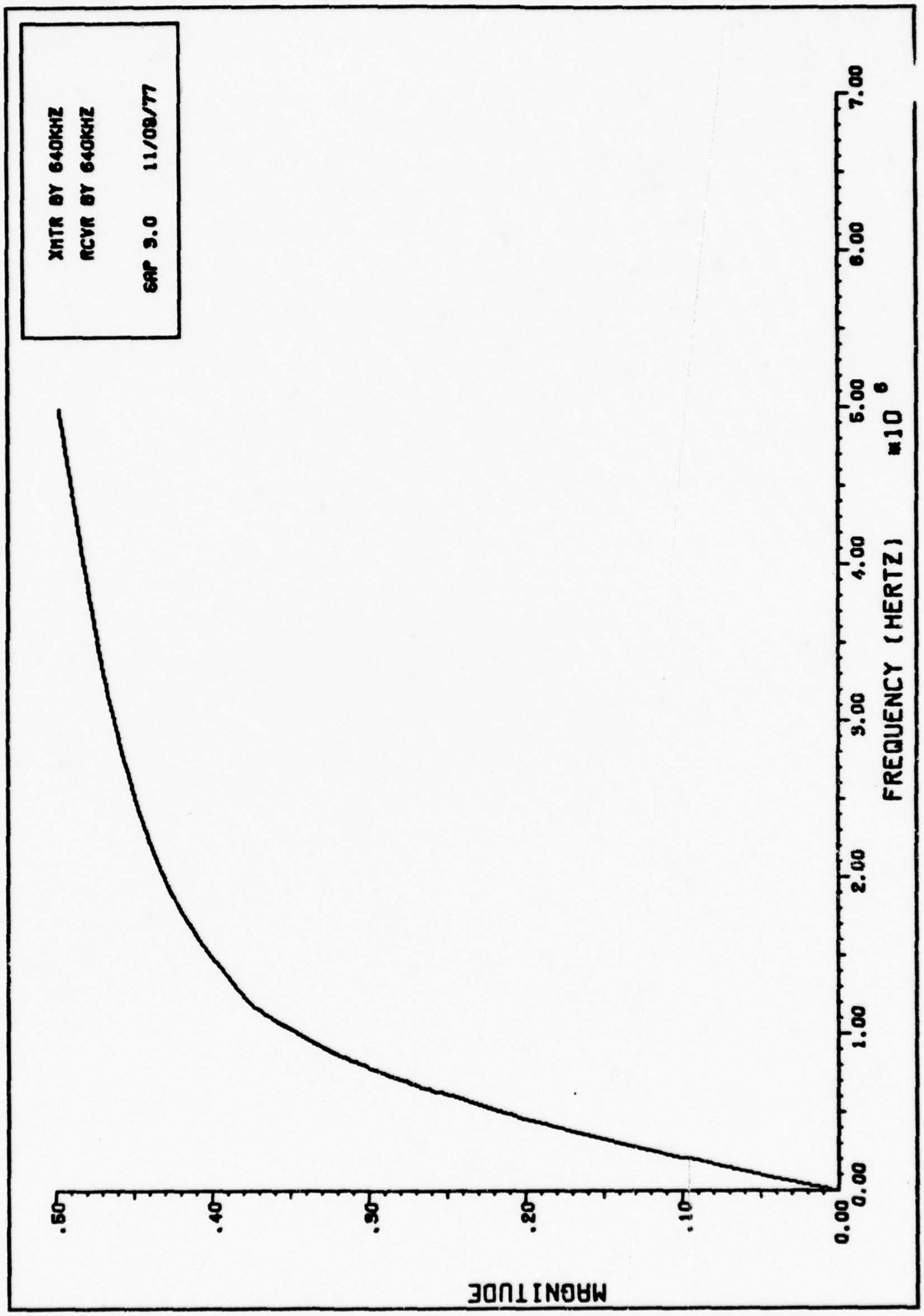

FIGURE 25. RECEIVED POWER VS RECEIVER BANDWIDTH

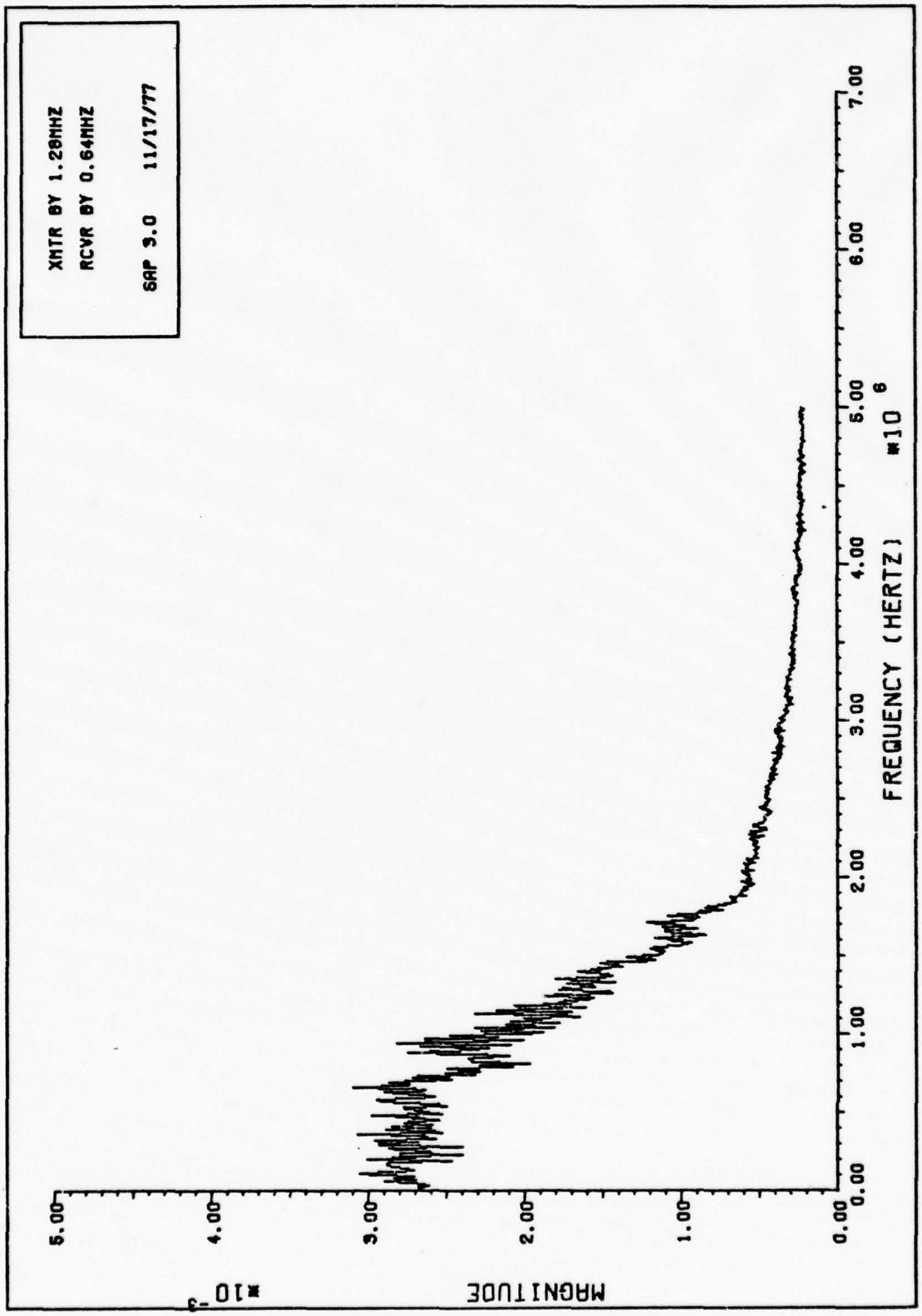

FIGURE 26. JOINT SPECTRUM

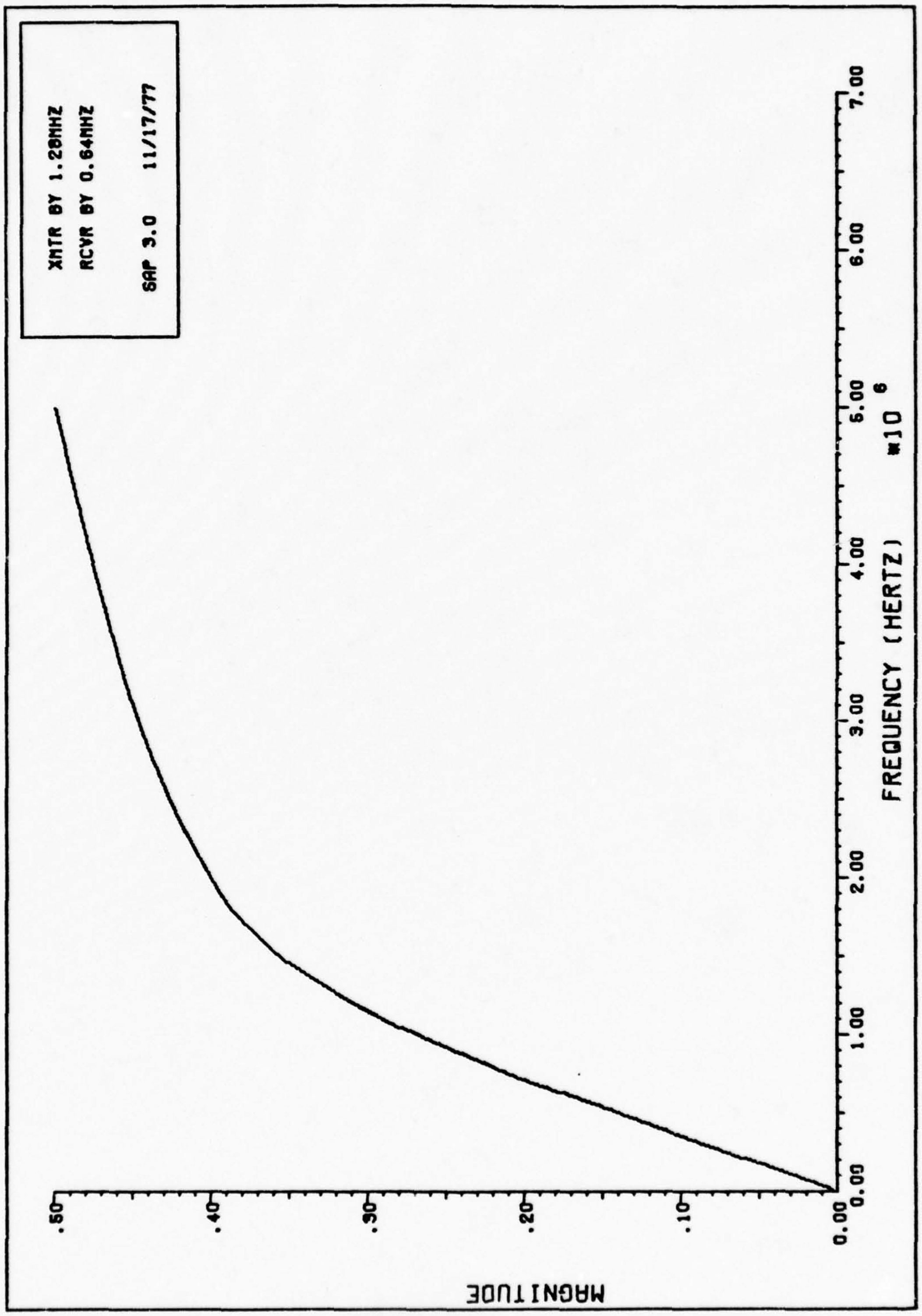

FIGURE 27. RECEIVED POWER VS RECEIVER BANDWIDTH

joint spectral density functions and the expected value of the received power are given respectively for all three cases of jammer filtering values, 320KHz, 640KHz, and 1.28MHz. For each case, the spectrum of the jammer process is convolved with the spectrum of the radar process to obtain the joint spectrum. The joint spectrums are spread due to the convolution. Figures 22, 24, and 26 demonstrate the gradual spread of the joint spectrum due to the increase of the spectral bandwidth of the jammer process. Figures 23, 25, and 27 illustrate the increasing dependence of the expected value of the received power upon the receiver bandwidth due to the increase of the spectral bandwidth of the jammer process. Thus, as was concluded in scenario (c), jamming effectiveness is reduced as the bandwidth of the jammer process becomes larger than the bandwidth of the receiver. However, the radar process as well as the jammer process dominate jamming effectiveness since both are playing the same role in creating the spectral behavior of the joint polarization diversity process.

V. Conclusion

The use of a polarization diversity process requires the derivation of a mathematical tool which will allow one to evaluate the effects of this process upon the received power. Partial derivation exists in the radio astronomy literature for a special case where only the transmitter employs a random process to produce the polarization diversity. Since random processes are assumed to be used for polarization diversity, the expected value of the received power has to be a function of the statistical characteristics of the processes. The time average operation used in radio astronomy is not as indicative of the statistical characteristics of the processes at an instant of time as much as may be the ensemble average.

Any polarization state may be produced by combining the waves radiated by a pair of cross-dipole antennas. This knowledge was used to construct a system model of polarization diversity. The mathematical dependence of the received power upon polarization diversity was derived by the use of this system model. The general transfer function of the received power was introduced first in the time domain. This function included the behaviors of the transmitter and the receiver by using their time domain transfer functions. The transfer function of the received power was transformed into the frequency domain where the received power was measured by its expected value. The likelihood of reception was introduced as a function of the spectral behavior of the polarization diversity process and the spectral characteristics of the transmitter and the receiver. The spectral

characteristic of the total polarization diversity process is constructed by the convolution between the polarization diversity processes of both the transmitter and receiver systems. The frequency spectrum of the polarization diversity is spread in a convolution manner by the transmitter frequency spectrum. The convolved frequency spectrum contributes to the expected value of the received power only within the bandwidth of the receiving system.

Two opposite approaches exist in optimizing the likelihood of reception of interference, maximization and minimization depending whether the ECM or the ECCM problem is considered, respectively. The optimizations factors of the polarization diversity process are first the distribution of spectral behavior of the process with respect to the receiver bandwidth and second the amplitude of the spectral behavior.

Four feasible types of scenarios were evaluated with respect to the system model of polarization diversity. The first two scenarios, consisting of random variables, were actually resolved by the use of the Poincare sphere representation of polarization states. Results of the last two scenarios, involving random processes, were obtained analytically and numerically. In both methods of analysis it was shown that by controlling the rate of change or equivalently the spectral behavior of the random processes employed by both the transmitter and receiver polarization diversity systems one could achieve either maximization or minimization of the likelihood of reception.

There is a definite disadvantage to the radar system if a random variable type of polarization diversity technique is employed by a jammer when the polarization state of the radar system is fixed. If

the polariation state of the radar system is controlled to nullify jammer interference by polarization orthogonality, the polarization diversity technique employed by the jammer is even more crucial and should be chosen to be a random process rather than a random variable. This essentially guarantees optimal likelihood of reception by the radar. In the case where the polarization state of the radar is a random process used to reduce jamming effectiveness in the manner of spread spectrum, the action that should be taken by the jammer is to correlate its process to the radar process. This action compresses the spectral behavior of the total polarization diversity process in a manner similar to the spectral compression of the radar signal.

This thesis did not take into consideration the problems one might face when choosing a polarization diversity process for either the jamming or the receiving systems. These problems are to retain the effectiveness of existing ECM or ECCM techniques while optimizing the likelihood of reception of the jamming signal by the radar and to develop a simple and feasible system to actually accomplish the polarization diversity. These problems are suggested for further study.

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Appendix A

Derivation of Equation 33

The purpose of this appendix is to show the detailed derivation of Equation 33. Some brief derivations have been written in the literature (Ref 1). Considering the vector potential at a point p, with a distance R from the origin of a radiating system, as illustrated in Figure A-1, one obtains the equation

$$\bar{A} = \int_{V'} \frac{\bar{i}_a \, e^{-jkr''}}{4\pi r''} \, dv' \tag{A-1}$$

where $\bar{A}$ is the phasor of the vector potential $\bar{\mathcal{A}}$ such that

$$\bar{\mathcal{A}} = \bar{A}e^{j\omega t} \tag{A-2}$$

In the far field, when $R \gg r'$, the following assumptions can be made. For amplitude consideration the assumption is

$$r'' \simeq R \tag{A-3}$$

but for phase consideration

$$r'' \simeq R - r'\cos\psi \tag{A-4}$$

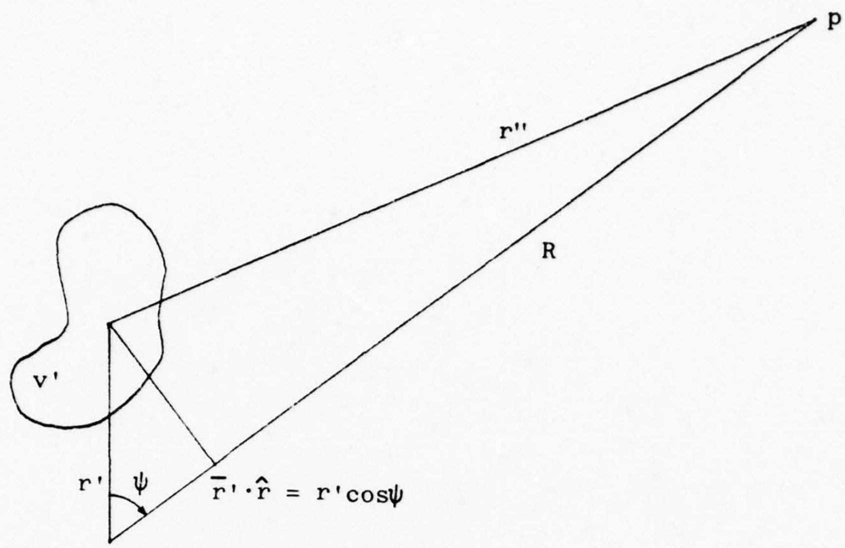

Figure A-1. Geometry of a Radiating System

where

$$\cos\psi = \cos\theta\cos\theta' + \sin\theta\sin\theta' \cos(\phi-\phi') \qquad \text{(A-5)}$$

Thus

$$\bar{A} = \frac{e^{-jkR}}{4\pi R} \int_{v'} \bar{i}_a e^{jkr'\cos\psi} \, dv' \qquad \text{(A-6)}$$

The integral is a function of the antenna configuration, current distribution, and direction of the current. This integral is defined as the radiation vector $\bar{N}$ such that

$$\bar{N} = \int_{v'} \bar{i}_a e^{jkr'\cos\psi} \, dv' \qquad \text{(A-7)}$$

Thus

$$\overline{A} = \frac{e^{-jkR}}{4\pi R} \overline{N} \tag{A-8}$$

It is convenient to describe the radiation vector as a multiplication of the input current I_i and an effective height vector of the antenna $\overline{h}$. For a thin linear antenna, this becomes

$$\overline{N} = \int_{l'} \int_{s'} \overline{i}_a e^{jkr'\cos\psi} ds' dl'$$

$$= \int_{l'} \left[\int_{s'} i_s(s') ds' \right] \overline{f}(l') e^{jkr'\cos\psi} dl' \tag{A-9}$$

or

$$\overline{N} = I_i \int_{l'} \overline{f}(l') e^{jkr'\cos\psi} \, dl'$$

$$= I_i \overline{h} \tag{A-10}$$

where s' is a surface intersecting the feed, $\overline{f}(l')$ is the current behavior along the length,

$$\overline{h} = \int_{l'} \overline{f}(l') e^{jkr'\cos\psi} \, dl' \tag{A-11}$$

and

$$\overline{A} = \frac{e^{-jkR}}{4\pi R} I_i \overline{h} \tag{A-12}$$

In a source-free medium, the plane wave form of the far field is

$$\overline{H} = \overline{H}_o e^{-j\overline{k}\cdot\overline{r}} \tag{A-13}$$

$$\overline{E} = \overline{E}_o e^{-j\overline{k}\cdot\overline{r}} \tag{A-14}$$

where $\overline{k} = k\hat{r}$ and $\overline{H}_o$ and $\overline{E}_o$ are constant vectors. Thus,

$$\overline{E} = \frac{1}{j\omega\varepsilon} \nabla \times \overline{H} \tag{A-15}$$

and when neglecting terms which decrease faster than 1/R in the far field

$$\overline{E} \simeq \frac{1}{j\omega\varepsilon} (-j\overline{k}\times\overline{H}) \tag{A-16}$$

Since

$$\overline{H} = \nabla \times \overline{A} \tag{A-17}$$

and

$$\overline{A} = A_o e^{-j\overline{k}\cdot\overline{r}} \tag{A-18}$$

therefore

$$\overline{H} \simeq -j\overline{k}\times\overline{A} \tag{A-19}$$

and

$$\overline{E} \simeq -\frac{k^2}{j\omega\epsilon}\,\hat{r} \times \hat{r} \times \overline{A} \tag{A-20}$$

or

$$\overline{E} \simeq j\omega\mu\,\hat{r} \times \hat{r} \times \overline{A}$$

$$= -j\omega\mu\,\overline{A}' \tag{A-21}$$

where

$$\overline{A}' = A_\theta\hat{\theta} + A_\phi\hat{\phi} \tag{A-22}$$

Replacing $\overline{A}'$ by $\overline{h}'$ according to Equation (A-12) leads to

$$\overline{E} \simeq -\frac{j\omega\mu}{4\pi R}\, e^{-jkr} I_i \overline{h}' \tag{A-23}$$

where

$$\overline{h}' = -\hat{r} \times \hat{r} \times \overline{h}$$

$$= h_\theta\hat{\theta} + h_\phi\hat{\phi} \tag{A-24}$$

with no r component. Thus

$$\overline{E} \simeq E_\theta\hat{\theta} + E_\phi\hat{\phi} \tag{A-25}$$

where

$$E_\theta = -\frac{j\omega\mu}{4\pi R}\, e^{-jkR}\, I_i h_\theta \tag{A-26}$$

and

$$E_{\phi} = -\frac{j\omega\mu}{4\pi R} e^{-jkR} I_i h_{\phi} \tag{A-27}$$

The poynting vector of the electric field has the form

$$\bar{p} = \mathrm{Re}[\bar{E} \times \bar{H}^*] \tag{A-28}$$

assuming rms magnitudes. The magnitude of the vector poynting is

$$p = \frac{1}{Z_o} [\bar{E} \cdot \bar{E}^*] \tag{A-29}$$

since the magnetic field is perpendicular to the electric field in a plane wave. Thus the magnitude of the poynting vector of an electromagnetic field produced by an antenna will be

$$p = \frac{1}{Z_o} \left(\frac{\omega\mu}{4\pi R} |I_i|\right)^2 (\bar{h}' \cdot \bar{h}'^*) \tag{A-30}$$

where $\bar{h}$ is the effective height of the antenna. Since

$$\begin{aligned} \frac{\omega\mu}{Z_o} &= \omega\sqrt{\mu\varepsilon} \\ &= \frac{2\pi}{\lambda} \end{aligned} \tag{A-31}$$

p becomes

$$p = Z_o\left(\frac{|I_i|}{2\lambda R}\right)^2(\bar{h}'\cdot\bar{h}'^*)$$

$$= Z_o\left(\frac{|I_i|}{2\lambda R}\right)^2(|h_\theta|^2+|h_\phi|^2) \tag{A-32}$$

Hence, the total power radiated by an antenna will be

$$P = \iint \bar{p}\cdot d\bar{s}$$

$$= \int_0^\pi \int_0^{2\pi} pR^2 d\Omega \tag{A-33}$$

where

$$d\Omega = \sin\theta d\theta d\phi \tag{A-34}$$

Thus

$$P = Z_o\left(\frac{|I_i|}{2\lambda}\right)^2 \int_0^\pi \int_0^{2\pi} (\bar{h}'\cdot\bar{h}'^*)d\Omega \tag{A-35}$$

The power radiated by the antenna can be expressed also by its feed current and effective radiation resistance. Thus, for rms magnitudes

$$P_t = R_{rad}|I_i|^2 \tag{A-36}$$

assuming no losses in the antenna, therefore the effective radiation resistance will be

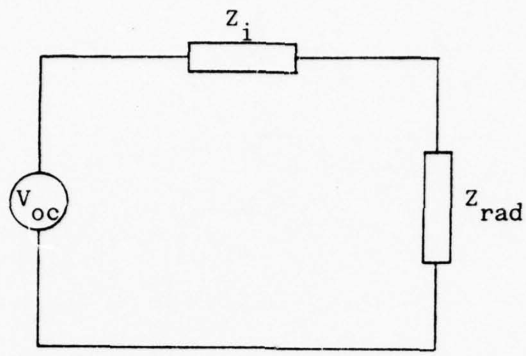

Figure A-2. Equivalent Circuit of an Antenna

$$R_{rad} = \frac{Z_o}{4\lambda^2} \int_0^{\pi} \int_0^{2\pi} (\bar{h}' \cdot \bar{h}'^*) d\Omega \tag{A-37}$$

The maximum power received by an antenna is reached when its internal impedance is equal to the conjugate impedance of the radiation impedance or that

$$R_{rad} = R_i \tag{A-38}$$

and

$$X_{rad} = -X_i \tag{A-39}$$

The model circuit of an antenna is illustrated in Figure A-2. Thus, the maximum power received by the antenna will be

$$P_r = \frac{v_{oc} v_{oc}^*}{4R_{rad}} \tag{A-40}$$

When the antenna is not matched the power will differ within a constant only. Sinclair has shown by using the reciprocity theorem that the open circuit voltage of a receiving antenna can be expressed as (Ref 6)

$$v_{oc} = \bar{E}_o \cdot \bar{h} \tag{A-41}$$

where $\bar{h}$ is the effective height vector of the receiving antenna and $\bar{E}_o$ is the incident electric field upon the antenna. Thus, substituting v_{oc} in Equation (A-40) yields

$$P_r = \frac{(\bar{E}_o \cdot \bar{h})(\bar{E}_o \cdot \bar{h})^*}{4R_{rad}} \tag{A-42}$$

Since in far field there is no r component of the incident field

$$P_r = \frac{1}{4R_{rad}} (E_\theta h_\theta + E_\phi h_\phi)(E_\theta h_\theta + E_\phi h_\phi)^* \tag{A-43}$$

The directive gain of an antenna is expressed by

$$D(\theta,\phi) = \frac{\bar{E} \cdot \bar{E}^*}{\frac{1}{4\pi} \iint_\Omega (\bar{E} \cdot \bar{E}^*) d\Omega} \tag{A-44}$$

If the antenna is 100% efficient, which means no mismatch or conductive losses, the gain and the directive gain are the same. If the antenna is not 100% efficient then the gain will be lower than the directive gain by

a factor which depends on the efficiency. Assuming a perfect antenna, the expression of the gain becomes then

$$G(\theta,\phi) = \frac{\bar{E}\cdot\bar{E}^*}{\frac{1}{4\pi}\iint\limits_{\Omega} (\bar{E}\cdot\bar{E}^*)d\Omega} \tag{A-45}$$

Since $\bar{h}'$ is relative to $\bar{E}$ by Equation (A-23), therefore the gain can be represented as a function of $\bar{h}'$

$$G(\theta,\phi) = \frac{\bar{h}'\cdot\bar{h}'^*}{\frac{1}{4\pi}\iint\limits_{\Omega} (h'\cdot h'^*)d\Omega} \tag{A-46}$$

Substituting Equation (A-46) in Equation (A-37) yields a simpler expression for R_{rad}, that is

$$R_{rad} = \frac{4\pi Z_o}{4\lambda^2}\,\frac{\bar{h}'\cdot\bar{h}'^*}{G(\theta,\phi)} \tag{A-47}$$

Substituting Equation (A-47) in Equation (A-43) yields

$$P_r = \frac{\lambda^2 G(\theta,\phi)}{4\pi}\, Z_o\, \frac{(\bar{E}_o\cdot\bar{h})(\bar{E}_o\cdot\bar{h})^*}{(\bar{h}'\cdot\bar{h}'^*)} \tag{A-48}$$

By multiplying the numerator and the denominator by the power density of the plane wave at the receiving antenna as in Equation (A-29) one obtains as a result the received power as

$$P_r(\theta,\phi,t) = \frac{\lambda^2 G(\theta,\phi)}{4\pi}\, p\, \frac{(\bar{E}_o\cdot\bar{h}')(\bar{E}_o\cdot\bar{h}')^*}{(\bar{E}_o\cdot\bar{E}_o{}^*)(\bar{h}'\cdot\bar{h}'^*)} \tag{A-49}$$

or

$$P_r(t) = A_e S \frac{|\bar{E}_o\cdot\bar{h}'|^2}{(\bar{E}_o\cdot\bar{E}_o{}^*)(\bar{h}'\cdot\bar{h}'^*)} \tag{A-50}$$

where

$$A_e = \frac{\lambda^2 G}{4\pi} \tag{A-51}$$

which is the effective aperture of the antenna and S is the real part of the poynting vector of the plane wave or the first Stokes parameter as in Equation (16). The components of the field and the effective height can be expressed as a magnitude and phase functions of time

$$\begin{aligned} E_\theta &= A_1(t)e^{-jkR} \\ &= |A_1(t)|e^{j[-kR+\alpha_1(t)]} \end{aligned} \tag{A-52}$$

$$\begin{aligned} E_\phi &= A_2(t)e^{-jkR} \\ &= |A_2(t)|e^{j[-kR+\alpha_2(t)]} \end{aligned} \tag{A-53}$$

and

$$\begin{aligned} h_\theta &= B_1(t) \\ &= |B_1(t)|e^{-j\beta_1(t)} \end{aligned} \tag{A-54}$$

$$h_\phi = B_2(t)$$

$$= |B_2(t)|e^{-j\beta_2(t)} \qquad \text{(A-55)}$$

where the minus sign in front of the $B_i(t)$'s is due to the opposite direction of the transmitting path from the receiving path. Thus, by substituting Equation (A-52) through (A-55) in Equation (A-50) one obtains

$$P_r = \frac{A_e S}{(|A_1(t)|^2+|A_2(t)|^2)(|B_1(t)|^2+|B_2(t)|^2)} \Big[|A_1(t)|^2|B_1(t)|^2$$

$$+|A_2(t)|^2|B_2(t)|^2$$

$$+|A_1(t)||A_2(t)||B_1(t)||B_2(t)|e^{j[\alpha_1(t)-\alpha_2(t)]}e^{-j[\beta_1(t)-\beta_2(t)]}$$

$$+|A_1(t)||A_2(t)||B_1(t)||B_2(t)|e^{-j[\alpha_1(t)-\alpha_2(t)]}e^{j[\beta_1(t)-\beta_2(t)]} \Big]$$

(A-56)

or by reordering the terms,

$$P_r = \frac{A_e S}{(|A_1(t)|^2+|A_2(t)|^2)(|B_1(t)|^2+|B_2(t)|^2)} \Big[\tfrac{1}{2}|A_1(t)|^2|B_1(t)|^2$$

$$+ \tfrac{1}{2}|A_1(t)|^2|B_2(t)|^2+\tfrac{1}{2}|A_2(t)|^2|B_1(t)|^2+\tfrac{1}{2}|A_2(t)|^2|B_2(t)|^2$$

$$+ \tfrac{1}{2}|A_1(t)|^2|B_1(t)|^2-\tfrac{1}{2}|A_1(t)|^2|B_2(t)|^2-\tfrac{1}{2}|A_2(t)|^2|B_1(t)|^2$$

$$+ \tfrac{1}{2}|A_2(t)|^2|B_2(t)|^2+\tfrac{1}{4}4\mathrm{Re}\Big(|A_1(t)||A_2(t)||B_1(t)||B_2(t)|e^{j[\alpha(t)-\beta(t)]}\Big)\Big]$$

(A-57)

where

$$\alpha(t) = \alpha_1(t) - \alpha_2(t) \tag{A-58}$$

and

$$\beta(t) = \beta_1(t) - \beta_2(t) \tag{A-59}$$

Since

$$\begin{aligned} \mathrm{Re}[e^{j[\alpha(t)-\beta(t)]}] &= \cos[\alpha(t) - \beta(t)] \\ &= \cos\alpha(t)\cos\beta(t) + \sin\alpha(t)\sin\beta(t) \end{aligned} \tag{A-60}$$

and by substituting Equations (10) through (13) and Equations (22) through (25) into Equation (A-57) one obtains

$$\begin{aligned} P_r &= \tfrac{1}{2}A_e S \; \frac{S_o a_o + S_1 a_1 + S_2 a_2 + S_3 a_3}{S_o} \\ &= \tfrac{1}{2}A_e S[\tilde{a}_i][s_i] \qquad , \; i = 0,\ldots,3 \end{aligned} \tag{A-61}$$

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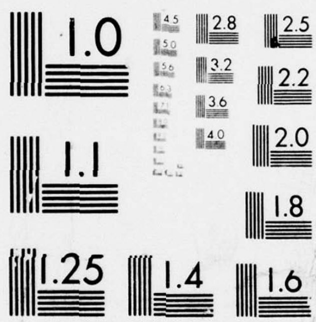

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Appendix B

Definitions and Equations in Random Fields

This appendix provides definitions and equations in random fields for the statistical terms used throughout the thesis. When further information considering these terms is needed one may refer to Papoulis (Ref 7).

Random Variables

A variable which is a function of the result of a statistical experiment, in which each outcome has a definite probability of occurrence, is called a random variable.

Probability Distribution Function. The probability distribution function, $F_X(x)$, of a random variable X is defined as the probability that $X \leq x$, that is

$$F_X(x) = P(X \leq x) \tag{B-1}$$

In case of an experiment with two random variables, X and Y, the joint probability distribution function is defined as

$$F_{XY}(x,y) = P(X \leq x,\ Y \leq y) \tag{B-2}$$

Probability Density Function. The probability density function of a random variable X, $f_X(x)$ or $f(x)$ for simplicity, is defined as

$$f_X(x) = \frac{dF_X(x)}{dx} \tag{B-3}$$

The joint probability density function is defined as

$$f_{XY}(x,y) = \frac{\partial^2 F_{XY}(x,y)}{\partial x \partial y} \tag{B-4}$$

Statistical Dependency. Two random variables, X and Y, are statistically independent if, for any x and y

$$P(X \leq x,\ Y \leq y) = P(X \leq x)\ P(Y \leq y) \tag{B-5}$$

or

$$f_{XY}(x,y) = f_X(x)\ f_Y(y) \tag{B-6}$$

If the two random variables are statistically dependent, the joint probability density function *will be*

$$\begin{aligned} f_{XY}(x,y) &= f_X(x)\ f_{Y|X}(y|x) \\ &= f_Y(y)\ f_{X|Y}(x|y) \end{aligned} \tag{B-7}$$

where $f_{X|Y}$ is the conditional probability density function of X given that Y is known and is defined as

$$f_{X|Y}(x|y) = \lim_{\Delta x \to 0} \frac{P[x-\Delta x < X \leq x \text{ given } Y=y]}{\Delta x} \tag{B-8}$$

and similarly for $f_{Y|X}(y|x)$.

Expected Value. The expected value $E\{X\}$, or the mean η, of a random variable X, is defined as

$$E\{X\} = \int_{-\infty}^{\infty} x\, f_X(x)dx \tag{B-9}$$

If X can have only discrete values x_n then

$$E\{X\} = \sum_n x_n p_n \tag{B-10}$$

where p_n is the respective probability of the value x_n. The expected value of a function of a random variable, $Y=g(X)$, is given by

$$E\{Y\} = \int_{-\infty}^{\infty} g(x) f_X(x)dx \tag{B-11}$$

or

$$E\{Y\} = \sum_n g(x_n)p_n \tag{B-12}$$

The expected value of a function of two random variables, $g(X,Y)$, will be

$$E\{g(X,Y)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)f_{XY}(x,y)dxdy \tag{B-13}$$

where $f_{XY}(x,y)$ is defined by Equations (B-6) or (B-7). The expected value of a linear combination of N random variables is equal to the same linear combination of their expected values. Thus,

$$E\left\{\sum_{i=1}^{N} a_i X_i\right\} = \sum_{i=1}^{N} a_i E\{X_i\} \tag{B-14}$$

Uncorrelated Variables. If X and Y are uncorrelated then

$$E\{XY\} = E\{X\}\, E\{Y\} \tag{B-15}$$

However, when $g(X,Y) = u(X)v(Y)$ and X and Y are uncorrelated, it does not necessarily follow that u and v are uncorrelated. But if X and Y are statistically independent then

$$E\{g(X,Y)\} = E\{u(X)\}\ E\{v(Y)\} \tag{B-16}$$

Orthogonality. Two random variables, X and Y, are called orthogonal if

$$E\{XY\} = 0 \tag{B-17}$$

Variance. The variance σ^2 of the random variable X, is defined by

$$\sigma^2 = E\{(X-\eta)^2\}$$

$$= \int_{-\infty}^{\infty} (x-\eta)^2\ f(X)dX \tag{B-18}$$

or

$$\sigma^2 = \sum_n (x_n - \eta)^2 \; p_n \tag{B-19}$$

when X has only discrete values x_n. The variance can be expressed by

$$\sigma^2 = E\{X\} - E^2\{X\}$$

$$= E\{X^2\} - \eta^2 \tag{B-20}$$

Random Processes

A random process is defined as a random variable in which to each outcome of the experiment a time function is assigned.

Stationary Processes. A random process is stationary in a strict sense if its statistics are not affected by a shift in the time origin. This means that the two processes $X(t)$ and $X(t+\varepsilon)$ have the same statistics for any ε. The two processes, $X(t)$ and $y(t)$, are jointly stationary if the joint statistics of $X(t)$, $Y(t)$ are the same as the joint statistics of $X(t+\varepsilon)$, $Y(t+\varepsilon)$ for any ε. The expected value of a stationary process $X(t)$ is a constant,

$$E\{X(t)\} = \eta$$

$$= \text{constant} \tag{B-21}$$

Correlation. For the purpose of this thesis, the autocorrelation of a stationary, complex process $X(t)$ is defined by

$$R_X(\tau) = E\{X(t+\tau)\, X^*(t)\} \tag{B-22}$$

assuming the real and the imaginary parts of X(t) to have the same statistics. The cross-correlation of two stationary processes, X(t) and Y(t), is defined as

$$R_{XY}(\tau) = E\{X(t+\tau)\ Y^*(t)\} \tag{B-23}$$

A process X(t) is stationary in a wide sense if its expected value is a constant and its autocorrelation depends only on $\tau = t_1 - t_2$. Two processes are jointly stationary in a wide sense if each one of them satisfies Equations (B-21) and (B-22) and their cross-correlation depends only on $\tau = t_1 - t_2$, as in Equation (B-23). In the following discussion, only stationary processes will be considered. Two processes X(t) and Y(t) are called uncorrelated if, for any τ

$$R_{XY}(\tau) = \eta_X\, \eta_Y^* \tag{B-24}$$

Covariance. The autocovariance of a process X(t) is defined as

$$C(\tau) = E\{(X(t+\tau) - \eta)(X^*(t) - \eta^*)\} \tag{B-25}$$

The cross-covariance of two processes X(t) and Y(t) is defined as

$$C_{XY}(\tau) = E\{(X(t+\tau) - \eta_X)(Y^*(t) - \eta_Y^*)\} \tag{B-26}$$

Thus,

$$C(\tau) = R(\tau) - |\eta|^2 \tag{B-27}$$

and

$$C_{XY}(\tau) = R_{XY}(\tau) - \eta_X \eta_Y^* \tag{B-28}$$

If $X(t)$ and $Y(t)$ are uncorrelated then, for any τ,

$$C_{XY}(\tau) = 0 \tag{B-29}$$

If $X(t)$ and $Y(t)$ are orthogonal then

$$R_{XY}(\tau) = 0 \tag{B-30}$$

Ergodicity. $X(t)$ is ergodic in the most general form if its time averages are equal to the ensemble averages (i.e., expected values).

Time Invariant System. Transformation of a process $X(t)$ by a time invariant system without memory leaves the statistics of the results $Y(t)$ similar to those of $X(t)$. The output of such a system will be stationary only if the input is stationary in the strict sense. The expected value of $Y(t) = g[X(t)]$ will be

$$E\{Y(t)\} = \int_{-\infty}^{\infty} g(x)\, f_x(x;t)dx \tag{B-31}$$

and the autocorrelation will be

$$\begin{aligned} R_Y(\tau) &= E\{Y(t+\tau)\, Y^*(t)\} \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x_1)\, g(x_2)\, f(x_1,x_2;\tau)dx_1 dx_2 \end{aligned} \tag{B-32}$$

where x_1 and x_2 are dummy variables, $f(x_1,x_2;\tau)$ is the joint density function defined as

$$f(x_1,x_2;\tau) = \frac{\partial^2 F(x_1,x_2;\tau)}{\partial x_1 \partial x_2} \tag{B-33}$$

and

$$F(x_1,x_2;\tau) = P\{X(t+\tau) \leq x_1,\ X(t) \leq x_2\} \tag{B-34}$$

<u>Power Spectral Density Function</u>

The power spectral density $S(\omega)$ of a process $X(t)$ is the Fourier transform of its autocorrelation, $R(\tau)$:

$$S(\omega) = \int_{-\infty}^{\infty} e^{-j\omega\tau} R(\tau) d\tau \tag{B-35}$$

$S(\omega)$ is a real and nonegative function. From the Fourier inversion formula follows that $R(\tau)$ can be expressed in terms of $S(\omega)$ by

$$R(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) e^{j\omega\tau}\, d\omega \tag{B-36}$$

With $\tau=0$, the above yields

$$\begin{aligned} \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) d\omega &= R(0) \\ &= E\{X(t)X^*(t)\} \end{aligned} \tag{B-37}$$

This is equal to the "average power" of the process $X(t)$. The cross-power

spectral density function $S_{XY}(\omega)$ of two processes $X(t)$ and $Y(t)$ is the Fourier transform of their cross-correlation:

$$S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau) e^{-j\omega\tau}\, d\tau \tag{B-38}$$

The Fourier inversion formula gives

$$R_{XY}(\tau) = \int_{-\infty}^{\infty} S_{XY}(\omega)\ e^{j\omega\tau}\, d\omega \tag{B-39}$$

and with $\tau=0$,

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XY}(\omega) d\omega = R_{XY}(0)$$

$$= E\{X(t)Y^{*}(t)\} \tag{B-40}$$

If the processes $X(t)$ and $Y(t)$ are orthogonal, then, due to Equation (B-30)

$$S_{XY}(\omega) = 0 \tag{B-41}$$

<u>Linear Systems</u>. The power spectral density $S_Y(\omega)$ of the output of a linear system with system function $H(j\omega)$ is given by

$$S_Y(\omega) = S_X(\omega)\ |H(j\omega)|^2 \tag{B-42}$$

where $S_X(\omega)$ is the power spectral density of the input. The linear system is illustrated in Figure (B-1).

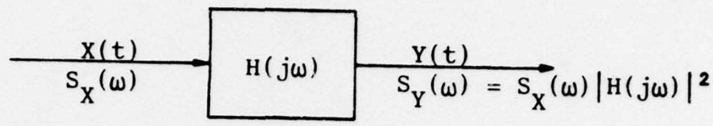

Figure B-1. The Linear System

Multiplication of Two Processes. The autocorrelation of the product $Z(t)$ of two random processes, $X(t)$ and $Y(t)$, can be expressed as

$$
\begin{aligned}
E\{Z(t)Z^*(t-\tau)\} &= E\{X(t)Y(t)X^*(t-\tau)Y^*(t-\tau)\} \\
&= E\{X(t)X^*(t-\tau)Y(t)Y^*(t-\tau)\} \\
&= E\{X(t)X^*(t-\tau)\}E\{Y(t)Y^*(t-\tau)\}+C_Z(\tau) \\
&= R_X(\tau)R_Y(\tau) + C_Z(\tau) \qquad \text{(B-43)}
\end{aligned}
$$

where $C_Z(\tau)$ is called the cross covariance and will be defined as

$$
\begin{aligned}
C_Z(\tau) &= E\{[X(t)X^*(t-\tau)-R_X(\tau)][Y(t)Y^*(t-\tau)-R_Y(\tau)]\} \\
&= E\{X(t)X^*(t-\tau)Y(t)Y^*(t-\tau)\} - R_X(\tau)R_Y(\tau) \qquad \text{(B-44)}
\end{aligned}
$$

The product is illustrated in Figure (B-2). If $X(t)$ and $Y(t)$ are statistically independent then

$$C_Z(\tau) = 0 \qquad \text{(B-45)}$$

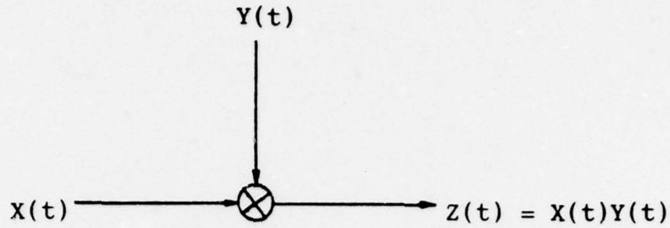

Figure B-2. The Product of Two Random Processes

From Equation (B-27),

$$R_X(\tau) = C_X(\tau) + |\eta_X|^2 \tag{B-46}$$

and

$$R_Y(\tau) = C_Y(\tau) + |\eta_Y|^2 \tag{B-47}$$

Thus

$$\begin{aligned} E\{X(t)Y(t)X^*(t-\tau)Y^*(t-\tau)\} &= R_X(\tau)R_Y(\tau) + C_Z(\tau) \\ &= [C_X(\tau)+|\eta_X|^2][C_Y(\tau)+|\eta_Y|^2] + C_Z(\tau) \\ &= C_X(\tau)C_Y(\tau) + |\eta_Y|^2 C_X(\tau) \\ &\quad + |\eta_X|^2 C_Y(\tau) + |\eta_X|^2|\eta_Y|^2 + C_Z(\tau) \end{aligned} \tag{B-48}$$

Vita

Meir Lavi was born on 22 Nisan 5705 (5 April 1945) in Jerusalem, Israel. He graduated from Yeshivat Haishuv-Hachadash High School, Tel-Aviv, Israel. Upon graduation he was enrolled at Israel Institute of Technology, Haifa, and in June 1968 was graduated with the degree of Bachelor of Science in electrical engineering. He joined the Israeli Air Force on 1 November 1968.

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The use of polarization diversity process requires the derivation of a mathematical system model to allow evaluating the effects of this process, employed by a jammer or by a threatened radar, upon jamming effectiveness. The system model consists of two orthogonal linear antennas controlled by a random process in amplitude and phase. The expected value of the received power due to interference is related to the autocorrelation of the received signal. The received power is obtained in the frequency domain from the spectral behavior of the total polarization diversity processes employed by both the jammer and

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radar and the spectral characteristics of the transmitting and receiving systems. The spectral behavior of the total polarization diversity process is constructed by the convolution between the polarization diversity processes of both the jammer and radar. The spectrum of the polarization diversity is spread in a convolution manner by the transmitter spectrum. This convolved spectrum contributes to the expected value of the received power only within the bandwidth of the receiving system. It is concluded that for several typical scenarios polarization diversity is an effective jamming technique.